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Department of Ocean Engineering
Cambridge, MA 02139

Report No. 78-4

POTENTIAL THEORY OF STEADY MOTION OF SHIPS, PARTS 1 AND 2

by

Francis Noblesse

September 1978

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A new theory of steady rectilinear motion of a ship in a calm sea is presented. This theory is based on a new integral equation for the velocity potential of the flow caused by the ship. Two outstanding features of the integral equation are that it is valid both on the hull surface (as usual) and in the flow domain, and that it immediately provides explicit approximations for the velocity potential. First and second approximations are given and discussed in some detail.

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POTENTIAL THEORY OF STEADY MOTION OF SHIPS, PARTS 1 AND 2

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ABSTRACT

A new theory of steady rectilinear motion of a ship in a calm sea is presented. This theory is applicable to displacement ships, fully-submerged bodies, multihull vessels, and surface-effect ships. It is based on a new integral equation for the velocity potential of the flow caused by the ship. The actual nonlinear free-surface boundary condition and the exact boundary condition at the ship hull, including effects of sinkage and trim, are incorporated into this integral equation, which also includes approximate ad hoc corrections for effects of viscosity, spray, and wavebreaking. Two outstanding features of the integral equation are that it is valid both on the hull surface (as usual) and in the fluid domain, and that it immediately provides explicit approximations for the velocity potential. First and second approximations are given. These new approximations generalize the traditional perturbation approximations; in particular, the classical first- (Michell) and second-order thin-ship approximations are obtained as "thin-ship limits" of these approximations. Of particular interest is the "initial approximation" given by equation (2.22); this new simple "linearized approximation" consists of a surface integral, which in fact corresponds to Hogner's wave resistance formula, and a line integral, which may be particularly important for blunt ship forms, and causes a drastic reduction in the wave resistance at low Froude number.

INTRODUCTION

The problem examined in the present study is that of steady rectilinear motion of a ship in a calm sea. The paper is divided into two parts.

In Part 1, the problem of steady motion of a ship is formulated, in dimensionless form, as a "generalized Neumann-Kelvin problem" in a "solution domain" bounded by the horizontal plane of the free surface of the undisturbed sea and some arbitrary "fictitious hull surface", where the free-surface and hull boundary conditions are enforced, respectively. The free-surface boundary condition is enforced on the undisturbed free surface rather than on the actual free surface for obvious reasons of mathematical simplicity, while the hull boundary condition is enforced on a "fictitious hull surface" rather than on the actual ship hull surface for the sake of generality, and in particular because this may be an advantageous method of accounting for the modifications in hull form associated with the sinkage and trim experienced by the ship, and also for systematically investigating effects of hull form modifications in the hull design process; the "fictitious hull surface" would then generally not differ much from the actual ship hull surface, and in particular may naturally be chosen to coincide with it. The exact forms of the "hull and free-surface conditions", that is including sinkage and trim, and free-surface nonlinearities, are used in this "generalized Neumann-Kelvin problem", which thus is an essentially "exact" formulation of the problem of steady motion of a ship; some approximate ad hoc corrections for effects of the viscous boundary layer and wake around and behind the ship, spray formation at the ship bow, and wavebreaking, are also included in the present potential-flow theory.

A main difficulty of the problem of steady motion of a ship resides in the free-surface boundary condition, which is nonlinear. An attempt to assess the importance of free-surface nonlinearities is thus made in this study. Specifically, a relatively simple method for experimentally assessing the nonlinear terms in the free-surface boundary condition is proposed, and this method is used to estimate free-surface nonlinearities at the first crest of the bow wave along the hull for the case of wedge-ended hull forms, for which experiments have been performed by Standing and Ogilvie; it is found that the magnitude of the disturbance velocity caused by these "ship" models varies between 17% and 33% of the speed of the model, and the error due to linearization of the free-surface condition varies between 10% and 26%, for entrance angles varying between 10° and 30°. These findings suggest that the free-surface boundary condition may be treated as "weakly nonlinear", which is indeed the fundamental assumption underlying the present theory.

It also appears that free-surface nonlinearities are weak even in the immediate vicinity of the ship bow.

In summary, the present formulation of the problem of steady motion of a ship as a "generalized Neumann-Kelvin problem" is a straightforward extension of the "Neumann-Kelvin model" in which (i) potential flow is assumed, (ii) the free-surface condition is linearized, and (iii) effects of sinkage and trim are neglected; this linearized Neumann-Kelvin model and the generalized formulation presented in this study are based upon the facts that for usual ships (i) the value of the Reynolds number is quite large (typically above 10^9) and viscosity effects are confined to a thin boundary layer except in a region of relatively limited extent at the stern of the ship, (ii) free-surface nonlinearities appear to be weak, and (iii) sinkage and trim are fairly small, at least for a broad class of ships operating at low or moderate values of the Froude number.

In <u>Part 2</u>, an integral equation for the velocity potential ϕ of the disturbance flow caused by the ship is obtained by using the methods of potential theory, specifically by using the classical Green identity (2.7) applied to the potential ϕ and the fundamental solution (Green function) $G(\vec{x}_0,\vec{x})$ appropriate for the problem. The basic properties of the fundamental function $G(\vec{x}_0,\vec{x})$ are discussed in some detail; in particular, attention is called to the fact that the function $G(\vec{x}_0,\vec{x})$ satisfies different equations and boundary conditions depending on whether $z_0 < 0$ or $z_0 = 0$, as it is shown explicitly in equations (2.5) and (2.6). No assumption or restriction is introduced in the course of the derivation of this new integral equation, which is therefore essentially equivalent to the "generalized Neumann-Kelvin problem" formulated in Part 1; in other words, the problem of steady motion of a ship is reformulated in "integral form", specifically given by the integral equation (2.14), equivalent to the "differential formulation" given in Part 1, namely the "generalized Neumann-Kelvin problem" stated by equations (1.14-1.16).

The integral equation (2.14) is the "key result" of the present study, and it is discussed in some detail. In the particular case of a surface-effect ship supported by an air cushion or a captive air bubble, the integral equation (2.14) takes the much-simplified form given by equation (2.15). An iterative method of solution of this integral equation for surface-effect ships is discussed; both the first and second "iterative" approximations are given explicitly. These "iterative" approximations are compared to the classical "perturbation" approximations based on the assumption of a "small free-surface pressure distribution", to which they are essentially equivalent, although not entirely identical.

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In the special case of a displacement ship, the integral equation (2.14) takes the form given by equation (2.21). With only few minor modifications, this integral equation for displacement ships is also valid for fully-submerged bodies and multi-hull vessels (catamaran, trimaran, SWATH). The integral equations (2.15) and (2.21) thus encompass most existing ships; however, problems associated with lift and cavitation for hydrofoils, and planing effects and spray formation for fast boats, are not considered in this study. Two outstanding features of the new integral equation (2.21) are (i) that it is valid not only on the ship hull surface (like the usual integral equations of potential theory) but also in the fluid domain outside the ship, as it is indeed necessary for incorporating the nonlinear terms in the free-surface condition, and (ii) that it immediately provides *explicit* approximations for the velocity potential ϕ .

Of particular interest is the "initial approximation" ϕ_I given by equation (2.22), which is obtained by merely ignoring all the unknown terms in the integral equation (2.21). This new simple "linearized approximation" ϕ_I consists of a surface integral, which in fact corresponds to the wave resistance formula proposed by Hogner [1] in 1932[+], and a line integral, which may be particularly important for blunt ship forms, and can be shown to cause a drastic reduction in the wave resistance at low Froude number. The second approximations ϕ_2 and ϕ_2' in the iterative schemes associated with the use of the potential ϕ_I and the Hogner potential ϕ_H [which are given by formulas (2.22) and (2.23), respectively] as initial approximations are given explicitly and discussed in some detail. These *iterative* approximations may be regarded as *generalizations* of the classical thin-ship *perturbation* approximations; indeed, both the first-order (Michell) and the second-order thin-ship approximations are rederived in the present study as "thin-ship limits" of the "fine-ship (iterative) approximations" ϕ_H and ϕ_2' .

It may finally be interesting to note in this Introduction that the second "fine-ship approximation" ϕ_2' given by formulas (2.30a) and (2.23) only involves surface distributions of sources — that is, neither a doublet distribution nor a line integral is involved — on the fictitious hull surface (h), which may (but need not) be taken as the submerged hull of the ship in position of rest, and on the undisturbed free surface (f). Such a representation of the potential ϕ in terms of surface source distributions on (h) and (f) can also be shown to be valid for the subsequent iterative approximations ϕ_k' , $k \geq 3$; this is indeed shown explicitly in the

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[+]I am indebted to Professor Som D. Sharma for pointing this fact out.

particular case of the "linearized Neumann-Kelvin problem" for which the solution ψ is expressed as the limit (assuming convergence) of the sequence of iterative approximations $\check{\psi}^{(k)}$, $k \geq 0$, defined by $\check{\psi}^{(0)} = 0$ and the recurrence relation (2.31), which only involves a surface source distribution on (h).

The single most important goal of any theory of steady motion of ships is undoubtedly the prediction of the drag, so-called "wave resistance", experienced by a ship as a result of its generating surface gravity waves. Exact expressions (to be sure, within the limitations of potential-flow theory) and various explicit approximations for the wave resistance of a displacement ship in steady motion, based on the new integral equation (2.21) obtained in Part 2 of this study, will be given in Part 3, which will be presented elsewhere.

Part 1: FORMULATION of the PROBLEM of STEADY MOTION of a SHIP as a GENERALIZED NEUMANN-KELVIN PROBLEM

Formulation of the hydrodynamical problem

In this section, we formulate the problem of steady, rectilinear motion, with speed U say, of a ship at the free surface of an otherwise calm sea assumed to be of infinite depth and lateral extent. Water is supposed to be homogeneous and incompressible; its density is denoted by ρ . Surface tension is neglected. Irrotational flow is assumed; some ad hoc corrections for viscosity effects, however, are allowed in this potential-flow formulation, as it will be seen below. The acceleration of gravity (the only body force considered) is denoted by g. The flow caused by the ship is rendered time independent by observing it from a system of coordinates moving along with the ship; the problem thus becomes that of determining the steady disturbance flow caused by a fixed ship hull in an oncoming uniform stream with velocity U .

A right-handed Cartesian system of coordinates (X,Y,Z) is defined, as follows: the (horizontal) plane of the undisturbed free surface is taken as the plane $Z=0$, and the Z axis (which is thus vertical) is chosen positive upwards; the X axis is parallel to the oncoming stream U , and pointing downstream (i.e. towards the stern of the ship). Naturally, in the (usual) case of a ship with a longitudinal plane of symmetry - referred to as the ship center-plane - moving without yaw or heel, so that the ship centerplane then is a vertical plane of symmetry for the flow, it is most convenient to take this centerplane as the plane $Y=0$. The origin of the X axis may be chosen at will, for instance, at the ship bow, or at midship.

A point in the flow domain, which is denoted by (D), then is defined by its position vector $\vec{X}(X,Y,Z)$. The flow domain is bounded by the free surface and the wetted hull of the ship, which are denoted by (F) and (H), respectively. The equation of (F) is taken as $Z=E(X,Y)$, where E thus represents the elevation of the free surface above (or below) the undisturbed level $Z=0$. The unit normal to (H), pointing inside the ship (i.e. outwards for the flow domain), is denoted by $\vec{N}$. The total fluid velocity may be expressed as $U\vec{i} + \nabla\Phi$, where $\vec{i}$ is a unit vector parallel to the oncoming uniform stream

(hence to the X axis) and in the same direction, and $\nabla\Phi$ is the disturbance flow velocity caused by the ship (the assumption of irrotational flow is evidently used here).

The "disturbance velocity potential" Φ must satisfy the Laplace equation in the flow domain (by the assumption of incompressibility). However, we will, more generally, allow a distribution of sources in (D), as a means for an ad hoc "viscosity correction". Justification of the introduction of such a distribution of sources for the purpose of representing the effects of the vorticity distribution in the boundary layer and wake around and behind the ship may be found in Landweber [2], where the relationship between vorticity and "equivalent" source distributions is examined. Here, we will merely point out that the replacement of the Laplace equation by a Poisson equation is "the best we can do" in the framework of the mathematical tools of potential-flow theory. Hence, the potential Φ is assumed to satisfy

$$\nabla^2\Phi = Q_D \text{ in } (D), \tag{1.1}$$

where Q_D is the strength of some unknown (but supposedly given as far as the potential-flow theory is concerned) source distribution in the ship's boundary layer and wake. Naturally, this source distribution need not be a volume distribution; for instance, it can be a distribution over some surface in (D).

The condition of no flow across the wetted hull surface is expressed by $(U\vec{i} + \nabla\Phi) \cdot \vec{N} = 0$ on (H). However, we will also allow a flux, Q_H say, across (H) as a (well-known) means of accounting for the "slowing down" of the velocity in the boundary layer. The "hull boundary condition" thus becomes

$$\nabla\Phi \cdot \vec{N} = -U\vec{i} \cdot \vec{N} - Q_H \text{ on } (H), \tag{1.2}$$

where $Q_H < 0$ evidently means fluid suction across (H).

The condition of no flow across the free surface is similarly somewhat "generalized" here by also allowing a flux, Q_F say, across (F). This might again be useful as a means of approximately accounting for viscosity effects; the "free-surface flux" Q_F can also serve to approximately represent the spray

which may often be observed at the bow of ships, particularly fast ones such as planing boats (the use of a line distribution of sinks along the waterline for the purpose of representing the effects of spray is suggested in Andersson [3]). The "kinematic free-surface boundary condition" then takes the form

$$\Phi_Z = (U + \Phi_X)E_X + \Phi_Y E_Y - (1 + E_X^2 + E_Y^2)^{\frac{1}{2}} Q_F \text{ on } (F) , \tag{1.3}$$

where $Q_F < 0$ means that fluid is sucked away across (F).

For most ships, the pressure at the free surface is constant, equal to the atmospheric pressure. However, a "free-surface pressure distribution" will be allowed here, mainly for the purpose of including the so-called "surface-effect ships" (supported by an air cushion or a captive air bubble); the "hull condition" (1.2) must evidently be ignored in this case of surface-effect ships. The Bernoulli equation then yields the "dynamic free-surface boundary condition"

$$U\Phi_X + \tfrac{1}{2}|\nabla\Phi|^2 + gE + P_F/\rho = 0 \text{ on } (F), \tag{1.4}$$

where P_F represents the deviation of the free-surface pressure from the atmospheric pressure (the fact that Φ, E and P_F vanish far upstream from the ship was evidently used here to determine the value of the "Bernoulli constant" in the Bernoulli equation). We may finally note here that the free-surface flux Q_F and pressure P_F in the free-surface conditions (1.3) and (1.4), respectively, could possibly also be used for the purpose of roughly representing the effects of wavebreaking.

Equations (1.1) - (1.4), to which we must add the usual radiation condition of no waves upstream from the ship, define the problem of steady motion of a ship. The unknowns are the disturbance velocity potential Φ, the free-surface elevation E, and also the position of the ship hull (H), which is not known exactly beforehand due to the unknown sinkage and trim experienced by the ship (as a result of departure of the pressure distribution at the hull from the hydrostatic rest distribution).

The ship's sinkage and trim may be determined from the equations stating the equilibrium of the ship as a rigid body acted upon by various forces, mainly its weight, buoyancy and hydrodynamic forces, and a propelling thrust of some kind.

The hydrodynamic/buoyancy forces consist of a lift and a moment, which are the main factors determining the ship's sinkage and trim (see, e.g., appendix C in Noblesse and Dagan [4]), as well as the ship's total resistance (due to wave-making, viscosity, wavebreaking and spray). The hydrodynamic lift and moment (and also the wave resistance) may be obtained by integration of the pressure distribution over the ship hull, and the pressure may evidently be obtained from the velocity potential Φ by means of the Bernoulli equation.

It is thus quite clear that the equations relating the ship's sinkage and trim to the potential Φ (in the manner described above) should be added to eqs. (1.1) - (1.4) and the radiation condition in order to have a closed set of equations, and hence a well-posed problem; this approach is indeed used in Peters and Stoker [5] and Wehausen [6]. However, we will rather incorporate effects of sinkage and trim by means of an iterative scheme, that is the "hydrodynamical problem", defined by eqs. (1.1) - (1.4) and the radiation condition, will be solved for an assumed hull surface (H); the solution Φ of this problem may then evidently be used to evaluate the sinkage and trim, and hence obtain a better approximation to (H), from which an improved approximation to Φ may be obtained, and so on.

In the case of a surface-effect ship, the free-surface pressure distribution P_F is similarly not known precisely beforehand, due to the influence of the shape of the free surface upon P_F, and the "hydrodynamical problem" [with the hull condition (1.2) being ignored] must also be coupled with additional equations for determining P_F precisely. An iterative scheme similar to that described above may be used in this case as well.

We may therefore focus our attention on the "hydrodynamical problem" defined by eqs. (1.1) - (1.4) and the radiation condition, where (H), P_F, Q_F, Q_H and Q_D are considered to be given. A fair estimate of the "viscosity hull flux" Q_H (at least over most of the hull surface where the boundary layer is thin) should be possible by present-day (three-dimensional turbulent) boundary-layer methods. The terms Q_F and Q_D, however, could only be determined empirically, and clearly must be regarded as means for attempting to approximately, and empirically, take into account the effects of the separated boundary layer and the wake at the stern of the ship, spray formation at the bow, and possibly also wavebreaking.

Formulation of the problem in dimensionless form

The first step in the investigation of the "hydrodynamical problem" should perhaps be to reformulate it in nondimensional form. All flow variables are made dimensionless in terms of the speed of the ship U, the acceleration of gravity g and the density of water ρ. Thus, we define

$$\left.\begin{aligned} &\nabla\phi = \nabla\Phi/U\,^{\dagger}, \quad \vec{x} = g\vec{X}/U^2\,, \quad e = gE/U^2\,, \\ &\phi = g\,\Phi/U^3\,, \quad p_F = P_F/\rho U^2\,, \\ &q_H = Q_H/U, \; q_F = Q_F/U\,, \; q_D = Q_D\,U/g\,. \end{aligned}\right\} \tag{1.5}$$

In terms of the above dimensionless variables, eqs. (1.1) - (1.4) take the form

$$\nabla^2\phi = q_D \text{ in } (D)\,, \tag{1.6}$$

$$\nabla\phi\cdot\vec{N} = -\vec{i}\cdot\vec{N} - q_H \quad \text{on } (H)\,, \tag{1.7}$$

$$\phi_z = (1+\phi_x)\,e_x + \phi_y\,e_y - (1+e_x^2+e_y^2)^{\frac{1}{2}}\,q_F \quad \text{on } (F)\,, \tag{1.8}$$

$$\phi_x + \tfrac{1}{2}|\nabla\phi|^2 + e + p_F = 0 \quad \text{on } (F). \tag{1.9}$$

The free-surface condition (1.9) may be used to determine the free-surface elevation e, and therefore the equation of (F); we readily obtain

$$z = e(x,y) = -\,\phi_x - \tfrac{1}{2}\,|\nabla\phi|^2 - p_F\,, \tag{1.10}$$

where it will be noted that the expression on the right side is evidently to be evaluated at the free surface, so that eq. (1.10) is in fact an implicit equation for (F) in terms of the potential ϕ.

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† Here, the convention is made that the differential operator ∇ is defined as $(\partial_X,\partial_Y,\partial_Z)$ when acting upon Φ, as in the previous section, while $\nabla \equiv (\partial_x,\partial_y,\partial_z)$ when applied to ϕ, that is hereafter in the paper.

By using equation (1.10) to eliminate the free-surface elevation e in equation (1.8), the boundary condition at the free surface may be expressed in terms of the potential ϕ alone; we obtain

$$\phi_z + \phi_{xx} + (|\nabla\phi|^2)_x + \tfrac{1}{2}\nabla\phi\cdot\nabla|\nabla\phi|^2 = -p^F_x - \phi_x p^F_x - \phi_y p^F_y - q_F' \quad \text{on (F)} ,$$

$$\text{that is on } z = -\phi_x - \tfrac{1}{2}|\nabla\phi|^2 - p_F , \qquad (1.11)$$

where the notation $p^F \equiv p_F$ was used for simplicity and q_F' is defined as

$q_F' = |\nabla(z + p_F + \phi_x + \frac{1}{2}|\nabla\phi|^2)| q_F$,

as it may be verified. Inasmuch as q_F merely represents a correction function to be determined empirically, we may evidently simplify condition (1.11) by assuming q_F' , rather than q_F , to be given (it will be noted that we have $q_F' = q_F$ in the linearized approximation).

The "hydrodynamical problem" of steady motion of a ship then consists in finding the disturbance velocity potential ϕ satisfying the Poisson equation (1.6), the Neumann condition (1.7), the free-surface condition (1.11), and the radiation condition.

It may be useful to comment here on the choice of U^2/g as reference length [see equations (1.5)] instead of the ship length, L say, as it is usually done.† The main reason for selecting the dimensionless variables (1.5) is clearly displayed in the resulting dimensionless equations (1.6) - (1.11), which may be seen to be free of any parameter [such as the Froude number $F = U/(gL)^{1/2}$, which appears in the free-surface condition (1.9), and hence (1.11), if L is taken as reference length]. It may easily be shown that the dimensionless variables (1.5) are the only ones which lead to dimensionless equations for the problem of steady motion of a ship that are free of any parameter,# and in this sense, the variables (1.5) may be said to be the "natural variables" for the problem. The most appreciable advantage of the above dimensionless formulation, however, may be that the fundamental solution (the Green function) then becomes a universal function (the parameter g/U^2, usually denoted by k_0 or K, which generally appears in the expression for the fundamental function, may simply be set equal to unity). This evidently makes it far easier for one to become acquainted with the (fairly in-

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†It should however be noted that the dimensionless variables (1.5) have of course been used previously, e.g. in Eggers, Sharma and Ward [7], and Standing [8].

#Naturally, the Froude Number F appears as a parameter via the size of the ship, which is inversely proportional to F^2 (so that "fast ships" are small, and "slow" ones are big).

tricate) behavior of this fundamental solution, which may be helpful, in particular for the purpose of numerical calculation.

There is another aspect of the choice of U^2/g as reference length which may perhaps also be discussed here. Flow past a ship hull is characterized by the superposition, and interaction, of two essentially different flow mechanisms. On the one hand, a ship hull, just like any other body placed in an oncoming uniform stream, represents an obstruction to the stream, so that we may expect regions of comparatively strong deceleration and acceleration in the vicinity of the bow and stern of a ship (except for idealized hull forms with cusped ends). A characteristic length of these regions of strong deceleration and acceleration may be taken as half the beam $B/2$ or the draft D, which are roughly equal for usual ship forms. On the other hand, the presence of the free surface causes the flow disturbance created by the ship (particularly at the bow and stern) to be propagated away from the ship in the form of surface waves, much like the oscillations caused by an initial displacement of a simple pendulum from its equilibrium position. The oscillatory flow in the ship waves mainly results from the mutual exchange between potential energy (characterized by the gravity g) and kinetic energy (characterized by U), and a characteristic length for this oscillatory flow may be taken as U^2/g. Indeed, the wavelength, λ_T say, of the transverse waves along the track of the ship is approximately given by $2\pi U^2/g$. Therefore, we have $U^2/g \simeq \lambda_T/6$, which shows that U^2/g may be regarded as a significant characteristic length for the flow acceleration associated with the oscillatory free-surface motion in the vicinity of the ship. The flow in the wave pattern at a certain distance away from the ship is relatively little affected by the ship itself, and U^2/g obviously is the basic characteristic length there.

Two characteristic lengths may thus essentially be distinguished, namely $D \simeq B/2$ and U^2/g. The ratio of these characteristic lengths is $U^2/gD \simeq U^2/(gB/2) \simeq F_T^2$, where $F_T \simeq U/(gD)^{\frac{1}{2}} \simeq U/(gB/2)^{\frac{1}{2}}$ is the Froude number based on the transverse dimensions of the ship. In the "moderate-Froude-number" range, i.e. for values of the "transverse Froude number" F_T which are neither too small nor too large in comparison with unity, it is evident that the selection of U^2/g as reference length is certainly well justified. The value of the Froude number F based on the length L of the ship, i.e. $F = U/(gL)^{\frac{1}{2}}$, usually varies between .1 and .5, while L/D and $L/(B/2)$ usually vary between 10 and 30; this yields $.3 < F_T < 3$. The value of the "transverse Froude number" F_T thus appears to be of order unity, so that the selection of U^2/g as reference length seems appropriate.

Investigation of free-surface nonlinearities

A readily apparent difficulty of the "hydrodynamical problem" formulated in the previous section stems from the free-surface condition (1.11), which is nonlinear, and holds on an unknown surface. This difficulty, however, is actually not as serious as it may perhaps appear at first sight, for condition (1.11) is "weakly nonlinear" almost everywhere in (F), that is except possibly in small regions in the neighborhood of the ship bow and stern; by "weakly nonlinear" is meant that the nonlinear terms in eq. (1.11) are sufficiently small compared with the linear ones that they may be neglected in a first approximation, and then taken into account by means of an iterative procedure (or a perturbation scheme) as "nonlinear corrections" to the "linearized approximation". The above assertion that the free-surface condition is weakly nonlinear almost everywhere in (F) will now be justified.

It may be instructive to begin by briefly considering potential flow past a fully-wetted body in an unbounded fluid (no free surface). It is well known that the dimensionless disturbance velocity $\nabla\phi$ ($\equiv \nabla\Phi/U$) caused by a body placed in an unbounded oncoming uniform stream (with velocity U) diminishes rapidly (like the inverse of the cube of the distance from the body) away from the body, so that $|\nabla\phi| << 1$ everywhere in the flow field, except close to the body. In fact, in the case of a slender body moving parallel to its major dimension, the regions of main flow deceleration and acceleration are confined to the immediate neighborhoods of the forward and rearward stagnation points, so that $\nabla\phi$ is small compared with unity everywhere in the flow field, even at the body surface, except in relatively small regions, approximately of the size of the transverse dimensions of the body, fore and aft of the body. For instance, the "longitudinal" disturbance velocity ϕ_x at the surface of a thin, two-dimensional body, of thickness/chord ratio ε , is approximately ε along the major (middle) portion of the body, that is except close to the fore and aft stagnation points (we have exactly $\phi_x = \varepsilon$ at the midsection of an elliptical cylinder). The corresponding result for a slender body, of slenderness parameter ε ($\equiv$ characteristic transverse dimension/length), is $\phi_x \simeq -\varepsilon^2 \ln\varepsilon$ [we have $\phi_x = -\varepsilon^2 \ln\varepsilon + O(\varepsilon^2)$ at the midsection of a prolate ellipsoid in axial motion].

One would not expect the above (evidently well-known) results to be grossly invalidated by the presence of a free surface, so that we might readily anticipate the assertion that the free-surface condition (1.11) is weakly nonlinear in (F)

except perhaps in relatively small regions around the ship bow and stern, to be justified. Gravity may nonetheless significantly alter the flow at the free surface (as it is attested to by the fact that the disturbance velocity in the wave pattern trailing behind the ship decreases like the inverse of the square root of distance, instead of the inverse of the cube of distance in the absence of a free surface), and a direct assessment of the importance of free-surface nonlinearities - based on experimental evidence - would then be proper. As a matter of fact, such an experimental investigation was already performed by Kitazawa, Inui and Kajitani [9] who carried out measurements of three-dimensional flow velocity components around the Inuid ship model M21 [$F = .29$, $L/(B/2) = 16.7$, $L/D = 11.6$]. On the basis of these actual flow velocity measurements, the authors concluded that the linear approximation to the free-surface boundary condition was well established, except perhaps at the first crest of the bow wave along the hull. Additional experimental results would evidently be interesting. The flow velocity measurements carried out in [9] are rather elaborate and time consuming, however, and it may then be useful to suggest here a simpler experimental method of assessment of free-surface nonlinearities; the purpose of this method is merely to assess the importance of free-surface nonlinearities along the load waterline.

We consider the case of a displacement ship in steady motion without yaw and heel. The equation of the ship hull is expressed in the form $y = \pm f(x,z)$, where $f(\equiv g F/U^2)$ is dimensionless, like the other flow variables. The hull boundary condition (1.7) then becomes

$$\pm \phi_y = (1+\phi_x)f_x + \phi_z f_z \quad \text{on (H)} , \tag{1.12}$$

where the hull flux q_H has been neglected here for simplicity. Equations (1.8) and (1.9) - where q_F and p_F will also be neglected - and eq. (1.12) are three algebraic equations for the three components ϕ_x, ϕ_y and ϕ_z of the disturbance velocity at the free-surface profile at the hull, i.e. along the curve defined by $y = \pm f(x,z)$, $z = e(x,y)$. By solving these equations, we may obtain

$$1 + \phi_x = (1-2e)^{\frac{1}{2}}(1 \mp e_y f_z)\Big[1 + f_x^2 + e_x^2 \mp 2\, e_y\, f_z + 2\, f_x\, e_x (f_z \pm e_y) +$$
$$+ f_x^2\, e_y^2 + e_x^2\, f_z^2 + e_y^2\, f_z^2\Big]^{-\frac{1}{2}} , \tag{1.13a}$$

$$\phi_y = \pm(1+\phi_x)(f_x + e_x f_z)/(1 \mp e_y f_z) \;, \tag{1.13b}$$

$$\phi_z = (1+\phi_x)(e_x \pm f_x e_y)/(1 \mp e_y f_z) \;. \tag{1.13c}$$

Along the load waterline, the disturbance velocity $\nabla\phi$ can then directly be obtained from the variables f_x, f_z, e, e_x, and e_y, which could be measured relatively easily.

The above equations will actually now be used to estimate the disturbance velocity at the first crest (where $e_x \simeq 0$) of the bow wave along the hull in the experiments on flow past vertical ($f_z = 0$) wedges reported by Standing [8] and Ogilvie [10]. By putting $f_x = \tan\alpha$, with α denoting the wedge half angle, $f_z = 0$, $e_x = 0$, and $e_y = 0$ (the free surface at the crest of the bow wave is thus supposed here to intersect the vertical hull surface perpendicularly) into eqs. (1.13), we easily obtain the longitudinal and lateral components, ϕ^*_x and ϕ^*_y say, of the disturbance velocity at the first crest of the bow wave in terms of the wedge half angle α and the elevation e^* of the crest of the wave above the undisturbed level: we obtain the remarkably simple results

$$1 + \phi^*_x = (1 - 2e^*)^{½} \cos\alpha \;, \quad \phi^*_y = \pm\,(1 - 2e^*)^{½} \sin\alpha \;.$$

The crest elevation e^* was estimated from Fig. 5 p. 636 in [8] , where wave profiles are shown for $\alpha = 5^\circ$ and 10° (hull speed $U = 1.22$ m/sec, draft $D =$ .73 m), and from the last photograph ($\alpha = 15^\circ$, $U = 3.49$ m/sec, $D = .305$ m) of Fig. 4 p. 30 in [10] . The results are shown in the table below

α	e^*	$-\phi^*_x$	$\lvert\phi^*_y\rvert$	$\lvert\nabla\phi^*\rvert$	$½\lvert\nabla\phi^*\rvert^2$	$½\lvert\nabla\phi^*\rvert^2/e^*$
5°	.135	.15	.07	.17	.014	.10
10°	.215	.26	.13	.29	.04	.20
15°	.205	.26	.20	.33	.05	.26

where the magnitude $\lvert\nabla\phi^*\rvert$ and the kinetic energy $½\lvert\nabla\phi^*\rvert^2$ of the disturbance velocity, as well as the ratio $½\lvert\nabla\phi^*\rvert^2/e^*$ have also been shown.

The magnitude of the disturbance velocity at the first crest of the bow wave along the hull is then found to vary between 17% and 33% of the speed of the ship model, while the ratio $\frac{1}{2}|\nabla\phi^*|^2/e^*$, which represents the error due to neglect of the nonlinear term $\frac{1}{2}|\nabla\phi|^2$ in the free-surface condition (1.9), varies between 10% and 26%. These numbers seem to indicate that the free-surface condition may indeed be regarded as weakly nonlinear even at the first crest of the bow wave along the hull, although it must certainly be kept in mind that they have been obtained here for the case of wedge-ended hull forms with fairly small entrance angles. Additional experimental evidence would be useful.

It may be interesting to note that the elevation of the first crest of the bow wave along the hull for the cases of wedge-ended hull forms examined above is significantly less than 1/2, which is the maximum possible elevation of the free surface, as it is well known; indeed, the Bernoulli equation (1.9), where p_F is taken equal to zero, may be written in the form

$$2e = 1 - [(1 + \phi_x)^2 + \phi_y^2 + \phi_z^2] ,$$

which readily shows that $e \leq 1/2$, with $e = 1/2$ corresponding to a stagnation point $1 + \phi_x = \phi_y = \phi_z = 0$ at the free surface. Furthermore, we may note that the first crest of the bow wave in these cases (see Figure 5 in [8] and the photographs of Figure 3 in [10]) does not occur right at the bow, but at some distance downstream, so that the free-surface elevation right at the bow is actually much less than 1/2 , and free-surface nonlinearities would then be weak even in the immediate neighborhood of the ship bow for the cases just examined. The experimental wave profiles reported by Gadd [11] p. 381 Figure 6 for the case of a ship hull of normal commercial form (single-screw merchant vessel of the DTMB Series 57, with C_B = .70, L/(B/2) = 14, and L/D = 17.4) show similar results: it can be estimated from Figure 6 in [11] that $.25 \leq e^* \leq .28$ (for $.208 \leq F \leq .253$), and the first crest of the bow wave also appears to be located downstream from the bow. Similar results are shown also in Figure 5 p. 553 of [9] for the Inuid model M21 [L/(B/2) = 16.7, L/D = 11.6, F = .29, e^* = .29].

Formulation of an "equivalent problem" in a "solution domain"

The derivation of the solution of the "hydrodynamical problem" in Part two of the present paper is based on the use of an appropriate fundamental solution (Green function) defined in the lower-half space $z \leq 0$. It is thus necessary to replace the free-surface boundary condition (1.11), which holds on (F) of course, by an equivalent condition on the plane $z = 0$. The use of such an "equivalent free-surface condition" on the undisturbed free surface is evidently also advantageous due to the unknown position of (F). The hydrodynamical problem defined previously will then be replaced by an equivalent problem in the "solution domain" bounded by the ship hull and the plane $z = 0$.

Each approximation to the ship's sinkage and trim in the iteration process defines a new position of the ship hull, with a corresponding modification of the solution domain. It may be advantageous to avoid these successive modifications of the solution domain - which are somewhat cumbersome, and might in addition require excessive computing times in practice - by also replacing the hull boundary condition (1.7), which holds on (H) of course, by an "equivalent hull condition" on some fixed fictitious hull surface, (h) say; an obvious choice for (h) is the wetted hull of the ship in position of rest. The use of such an "equivalent hull condition" on a fictitious hull surface may also be a convenient means of investigating the effects of hull-form modifications in the "hull-design process", where (h) may then be taken as the "hull at preliminary design", about which small modifications are being sought (for the purpose of minimizing the ship wave resistance, say). The two above-mentioned motivations (sinkage and trim, and hull design) for using an equivalent hull condition on a fictitious hull (h) evidently imply that (h) would in all cases be a surface close to the actual hull surface {even though in principle (h) could be chosen somewhat arbitrarily; indeed, in the thin-ship theory, (h) is taken as the projection of (H) onto the ship centerplane $y = 0$, as it is discussed in detail in [4]}.

We thus are led to formulate an equivalent problem to the hydrodynamical problem in a solution domain, (d) say, bounded by the fictitious hull (h) and the portion of the plane $z = 0$ outside (h), which is denoted by (f) for easy reference. A rigorous approach to the formulation of such an equivalent problem in a solution domain consists in formally introducing a one-to-one mapping of the flow domain (D) onto the solution domain (d). Inasmuch as (d) does not differ very much from (D), as it was already noted, the mapping of (D) onto (d) would be a "near-identity mapping". The use of a mapping is not indispensable in

this case. Instead, the alternative approach based on the assumption that the potential ϕ can be continued analytically wherever it is not defined and is needed may be used. The mapping method, in the case of a near-identity mapping, and this more usual approach are actually equivalent, as it is shown in Joseph [12] and Noblesse and Dagan [4]. The approach based on analytic continuation is more expedient, and will then be used here.

The "<u>equivalent free-surface condition</u>" on (f) can be derived from condition (1.11) by means of a straightforward modification, namely

$$\phi_z + \phi_{xx} = - p_x^F - q \quad \text{on (f)}, \tag{1.14}$$

where $q(x,y)$ is defined as

$$q = \left[\phi_z + \phi_{xx} + (|\nabla\phi|^2)_x + \tfrac{1}{2}\nabla\phi\cdot\nabla|\nabla\phi|^2 + \phi_x p_x^F + \phi_y p_y^F\right]_F - (\phi_z + \phi_{xx})_f + q'_F \ , \tag{1.14a}$$

with (F) defined by $z = -\phi_x - \tfrac{1}{2}|\nabla\phi|^2 - p_F$, as it is given by eq. (1.11). The "equivalent free-surface flux" $q(x,y)$ at (f) thus includes both the "free-surface flux" q'_F , which may embody effects of spray, wavebreaking and viscosity, and the free-surface nonlinearities, stemming from both the nonlinear terms in eq. (1.11) and the fact that (F) differs from the plane $z = 0$. The formulation (1.14) of the equivalent free-surface condition on (f) is evidently based on the fact that the free-surface boundary condition is weakly nonlinear almost everywhere in (F), as it was shown in the previous section.

The "<u>equivalent hull condition</u>" on (h) is similarly obtained from eq. (1.7) by merely rewriting this equation in the form

$$\nabla\phi\cdot\vec{n} = -\vec{i}\cdot\vec{n} - q \quad \text{on (h)} \ , \tag{1.15}$$

where $\vec{n}$ is the unit normal to (h) — pointing inside — and q is defined as

$$q = (\vec{i} + \nabla\phi)_H \cdot \vec{N} - (\vec{i} + \nabla\phi)_h \cdot \vec{n} + q_H \ . \tag{1.15a}$$

The "equivalent hull flux" q thus includes both the "viscosity flux" q_H, which embodies effects of the viscous boundary layer on the external potential flow, and the correction associated with the difference between (H) and (h), due to sinkage and trim or hull-form modifications in the hull design process; evidently if $(h) \equiv (H)$ we merely have $q \equiv q_H$.

It will be noted that the same notation q was used to represent the "equivalent flux" on both (f) and (h); this notation should not cause any confusion however, since the precise meaning of the flux q is evident from the particular boundary condition where q appears. The expression $[\phi_z + \phi_{xx} + (|\nabla\phi|^2)_x + \frac{1}{2}\nabla\phi \cdot \nabla|\nabla\phi|^2 + \phi_x p_x^F + \phi_y p_y^F]_F$ in eq. (1.14a), and the term $(\nabla\phi)_H$ in eq. (1.15a), at points of (F) and (H), respectively, outside the solution domain (d) must evidently be evaluated by means of analytic continuation of ϕ outside (d); this may be accomplished in practice in various ways, for instance by using an appropriate formula for numerical extrapolation.

The "equivalent problem" in the "solution domain" then is defined by the "equivalent free-surface and hull conditions" (1.14) and (1.15), and the Poisson equation

$$\nabla^2\phi = q \text{ in (d)}, \tag{1.16}$$

where the same notation q was used also to represent the "domain sources" q_D associated with the thick boundary layer and the wake at the stern of the ship; the radiation condition of no waves upstream from the ship must finally be added.

In the case of a displacement ship $[p_F \equiv 0$ in eq. (1.14)], the problem defined by assuming q is zero in eqs. (1.14), (1.15) and (1.16), and taking (h) as the wetted hull of the ship in position of rest, is recognized as the "Neumann-Kelvin problem" (this appellation was suggested by Brard [13] on account of the fact that "the boundary condition on the hull is of Neumann's type, while the boundary condition on the free surface is that used by Lord Kelvin when, the first of all, he initiated the mathematical researches on ship waves"). The "equivalent problem" defined here is then referred to as a "generalized Neumann-Kelvin problem" in order to emphasize its obvious affinity with the "Neumann-Kelvin problem", and also to call attention to the various corrections embodied in the nonhomogeneous terms q on the right sides of equations (1.14-1.16), as well as to the fact that (h) may differ from (H).

It is implicit in the above formulation of the "equivalent problem" that the "correction terms" q on the right sides of equations (1.14 - 1.16) are to be treated as <u>known nonhomogeneous terms</u> for the purpose of solving this "equivalent problem"; the "generalized Neumann-Kelvin problem" defined above then must be solved by means of an iterative procedure of some kind.

Equations (1.14-1.16) readily suggest a "natural" first (linearized) approximation, $\phi^{(1)}$ say, to the disturbance potential ϕ : this is the approximation defined by merely ignoring the correction terms q . This first approximation $\phi^{(1)}$ would thus be the solution of the problem defined by

$$\nabla^2\phi^{(1)} = 0 \text{ in } (d) , \tag{1.17a}$$

$$\nabla\phi^{(1)} \cdot \vec{n} = - \vec{i} \cdot \vec{n} \text{ on } (h) , \tag{1.17b}$$

$$\phi_z^{(1)} + \phi_{xx}^{(1)} = - \left(p_F^{(1)}\right)_x \text{ on } (f), \tag{1.17c}$$

and the usual radiation condition. The term $p_F^{(1)}$ on the right side of equation (1.17c) is the (given) first approximation to the free-surface pressure p_F acting on (F) in the case of a surface-effect ship [the influence of (F) upon p_F is neglected in the approximation $p_F^{(1)}$]; the hull condition (1.17b) should naturally be ignored in the case of a surface-effect ship. In the case of a displacement ship, on the other hand, we evidently have $p_F^{(1)} \equiv 0$, while a natural choice for (h) is the wetted hull of the ship in position of rest; the first approximation $\phi^{(1)}$ then is recognized as the so-called "Neumann-Kelvin approximation".

The first approximation $\phi^{(1)}$ could then be used to generate a sequence of approximations, $\phi^{(k)}$ ($k \geq 2$) say, by solving the sequence of problems defined by

$$\nabla^2 \phi^{(k)} = q^{(k-1)} \text{ in } (d) , \tag{1.18a}$$

$$\nabla \phi^{(k)} \cdot \vec{n} = - \vec{i} \cdot \vec{n} - q^{(k-1)} \text{ on } (h) , \tag{1.18b}$$

$$\phi_z^{(k)} + \phi_{xx}^{(k)} = - \left(p_F^{(k)}\right)_x - q^{(k-1)} \text{ on } (f) , \tag{1.18c}$$

and the radiation condition, where the correction terms $q^{(k-1)}$ are based on the previous approximations $\phi^{(1)}, \ldots , \phi^{(k-1)}$ to ϕ . In the case of a surface-effect ship, the approximation $p_F^{(k)}$ to the free-surface pressure p_F takes into account the influence of the free surface, the position of which is evidently determined from the previous approximations $\phi^{(1)}, \ldots , \phi^{(k-1)}$.

In summary, the problem of steady motion of a ship has been formulated above as a problem in partial differential equations - so-called "generalized Neumann-Kelvin problem" - defined by eqs. (1.14-1.16). The study of this problem is pursued in Part 2 of the present paper, where a new integral equation is formulated, and new explicit approximate solutions are obtained.

Part 2: INTEGRAL EQUATION and EXPLICIT APPROXIMATIONS for the VELOCITY POTENTIAL of the FLOW CAUSED by a SHIP in STEADY MOTION

Preliminary results concerning the fundamental solution

The proposed solution of the "generalized Neumann-Kelvin problem" is based on the use of an appropriate fundamental solution (Green function). This fundamental solution, which is usually denoted by $G(\vec{x},\vec{x}_0)$, is known to represent the linearized disturbance velocity potential at point $\vec{x}(x,y,z\leq 0)$ due to a fixed unit source at point $\vec{x}_0(x_0,y_0,z_0<0)$ in an oncoming uniform stream with unit velocity (it is emphasized that G, $\vec{x}$ and $\vec{x}_0$ are dimensionless as in Part 1 of the present paper). The fundamental solution $G(\vec{x},\vec{x}_0)$ thus is the solution of the problem defined by

$$\left.\begin{aligned} &\nabla^2 G = \delta(\vec{x}-\vec{x}_0) \quad \text{in } z < 0 , \qquad &(2.1a)\\ &G_z + G_{xx} = 0 \quad \text{on } z = 0 , \qquad &(2.1b)\end{aligned}\right\} \quad (z_0 < 0)$$

and the radiation condition, where $\delta(\vec{x}-\vec{x}_0)$ in equation (2.1a) represents the three-dimensional Dirac function defined as $\delta(\vec{x}-\vec{x}_0) = \delta(x-x_0)\ \delta(y-y_0)\ \delta(z-z_0)$. It is emphasized that z_0 in equations (2.1) is supposed to be strictly negative [although we may in fact let z_0 tend to zero in the expression for the <u>solution</u> $G(\vec{x},\vec{x}_0)$ of the foregoing problem]. We need not be concerned with the expression for the fundamental solution here, although we might note that a simplified new expression for $G(\vec{x},\vec{x}_0)$ was recently obtained in Noblesse [14], where a brief survey of known alternative expressions may also be found.

Let us now consider - for a moment - another fundamental solution, denoted by $G^*(\vec{x},x_0,y_0)$ say, representing the linearized disturbance velocity potential at point $\vec{x}(x,y,z\leq 0)$ due to a "unit flux" at point (x_0,y_0) on the undisturbed free surface $z = 0$ of an oncoming uniform stream with unit velocity (the same dimensionless variables are used here as previously). The linearized approximation to the free-surface condition (1.11) for the case of a fluid flux q_F across the free surface (but with $p_F = 0$) takes the form $\phi_z + \phi_{xx} = -q_F$. A "unit flux" across (F), or rather the plane $z = 0$ in the linear approximation, is defined by $q_F = \delta(x-x_0,\ y-y_0)$, where $\delta(x-x_0,\ y-y_0)$ represents the two-dimensional Dirac function defined as $\delta(x-x_0)\ \delta(y-y_0)$. The fundamental solution $G^*(\vec{x},x_0,y_0)$ then is the solution of the problem defined by

$$\nabla^2 G^* = 0 \quad \text{in } z < 0, \tag{2.2a}$$

$$G^*_z + G^*_{xx} = -\,\delta(x-x_0,\ y-y_0) \quad \text{on } z = 0, \tag{2.2b}$$

and the usual radiation condition.

From the physical significance of the fundamental solutions $G(\vec{x},\vec{x}_0)$ and $G^*(\vec{x},x_0,y_0)$ as the (linearized) disturbance velocity potentials associated with a "unit outflow" stemming from a submerged source at $(x_0,y_0,z_0 < 0)$ and a flux across the undisturbed free surface at $(x_0,y_0,z_0 = 0)$, respectively, it is intuitively evident that $G(\vec{x},\vec{x}_0) \to G^*(\vec{x},x_0,y_0)$ as $z_0 \to 0$. We thus expect

$$G^*(\vec{x},x_0,y_0) \equiv G(\vec{x},x_0,y_0,z_0 = 0) \ , \tag{2.3}$$

as it can actually be verified from the expressions for the solutions $G(\vec{x},\vec{x}_0)$ and $G^*(\vec{x},x_0,y_0)$ of the problems defined by eqs. (2.1) and (2.2), respectively (this verification may be found in Andersson [3]). It then follows from eqs. (2.2) and (2.3) that $G(\vec{x},x_0,y_0,z_0 = 0)$ must satisfy the equations

$$\left.\begin{aligned} &\nabla^2 G = 0 \quad \text{in } z < 0 \ , && (2.4a)\\ &G_z + G_{xx} = -\,\delta(x-x_0,\ y-y_0) \quad \text{on } z = 0 \ . && (2.4b)\end{aligned}\right\} \quad (z_0 = 0)$$

The fundamental solution $G(\vec{x},\vec{x}_0)$ was introduced above as the (linearized) disturbance velocity potential at point $\vec{x}$ due to a "unit outflow" at point $\vec{x}_0$ in an oncoming uniform stream with unit velocity along the x axis in the <u>positive</u> direction. However $G(\vec{x},\vec{x}_0)$ also represents the linearized disturbance velocity potential at point $\vec{x}_0$ due to a "unit outflow" at point $\vec{x}$ in an oncoming uniform stream with unit velocity along the x axis in the <u>negative</u> direction, as it is known (see, for instance, Brard [13]), and may be verified. The fundamental solution $G(\vec{x},\vec{x}_0)$ thus also satisfies the equations

$$\left.\begin{aligned}&\nabla_0^2 G = \delta(\vec{x}_0 - \vec{x}) \text{ in } z_0 < 0 ,\\ &G_{z_0} + G_{x_0x_0} = 0 \text{ on } z_0 = 0 ,\end{aligned}\right\} \quad (z < 0)$$

$$\left.\begin{aligned}&\nabla_0^2 G = 0 \text{ in } z_0 < 0 ,\\ &G_{z_0} + G_{x_0x_0} = -\delta(x_0 - x, y_0 - y) \text{ on } z_0 = 0 ,\end{aligned}\right\} \quad (z = 0)$$

where ∇_0 represents the differential operator $(\partial_{x_0}, \partial_{y_0}, \partial_{z_0})$ to distinguish it from the operator $\nabla \equiv (\partial_x, \partial_y, \partial_z)$; the foregoing equations evidently correspond to equations (2.1) and (2.4), respectively.

By interchanging the variables $\vec{x}$ and $\vec{x}_0$ in the above equations, it may then finally be seen that $G(\vec{x}_0, \vec{x})$, that is the linearized disturbance velocity potential at point $\vec{x}_0(x_0, y_0, z_0 \leq 0)$ due to a "unit outflow" (stemming from a submerged source or a flux across the free surface) at point $\vec{x}(x, y, z \leq 0)$ in an oncoming unit stream along the positive direction of the x axis, satisfies the equations

$$\left.\begin{aligned}&\nabla^2 G = \delta(\vec{x} - \vec{x}_0) \text{ in } z < 0 , &(2.5a)\\ &G_z + G_{xx} = 0 \text{ on } z = 0 , &(2.5b)\end{aligned}\right\} \quad (z_0 < 0)$$

$$\left.\begin{aligned}&\nabla^2 G = 0 \text{ in } z < 0 , &(2.6a)\\ &G_z + G_{xx} = -\delta(x - x_0, y - y_0) \text{ on } z = 0 , &(2.6b)\end{aligned}\right\} \quad (z_0 = 0)$$

It may be appropriate to point out here that - to the author's knowledge - all previous derivations of an integral equation for the present problem are based on equations (2.5) alone; it should however be realized that equations (2.5) are valid only for z_0 strictly negative, and that equations (2.6) hold for $z_0 = 0$. The derivation of the integral equation in the following section makes use of both equations (2.5) and (2.6).

Derivation of an integral equation from the generalized Neumann-Kelvin problem

We are now ready to derive an integral equation from the "generalized Neumann-Kelvin problem" defined by eq. (1.16), the boundary conditions (1.14) and (1.15), and the radiation condition. The "correction terms" q on the right sides of these equations are regarded here as arbitrary given functions.

The derivation of the integral equation is based on the use of a well-known Green identity, applied to the fundamental solution $G(\vec{x}_0,\vec{x})$ and the disturbance velocity potential $\phi(\vec{x})$, as follows

$$\int_{d'} (\phi\, \nabla^2 G - G\, \nabla^2\phi)dv = \int_h (\phi\, G_n - G\, \phi_n)\, da \; +$$

$$\int_{f'} (\phi\, G_z - G\, \phi_z)dxdy + \int_\sigma (\phi\, G_n - G\, \phi_n)\, da \;, \tag{2.7}$$

where (d') is the finite domain obtained by excluding the portion of (d) outside a certain arbitrary — but large enough to contain (h) entirely — bounding surface (σ), as it is shown in Fig. 1, (f') is the portion of (f) inside (σ), that is the region of the plane $z = 0$ between (h) and (σ), dv and da represent differential elements of volume and area of the domain (d') and its boundary (h) + (f') + (σ), $\vec{n}$ is the unit outward [with respect to (d')] normal to (h) or (σ), and the notation $\phi \equiv \phi(\vec{x})$, $G \equiv G(\vec{x}_0,\vec{x})$, $\nabla \equiv (\partial_x,\partial_y,\partial_z)$, $dv \equiv dv(\vec{x})$, $da \equiv da(\vec{x})$, $\vec{n} \equiv \vec{n}(\vec{x})$, $G_n \equiv \nabla G \cdot \vec{n}$, and $\phi_n \equiv \nabla\phi \cdot \vec{n}$ was used for shortness. By using eqs. (1.14-1.16) in eq. (2.7), we obtain

$$\int_{d'} \phi\, \nabla^2 G\, dv \; - \int_{d'} G\, q\, dv \; = \int_h \phi\, G_n\, da \; + \int_h G\, \nu\, da \; +$$

$$+ \int_h G\, q\, da \; + \int_{f'} (\phi\, G_z + G\, \phi_{xx})\, dxdy \; + \int_{f'} G\, p_x^F\, dxdy \; +$$

$$+ \int_{f'} G\, q\, dxdy \; + \int_\sigma (\phi\, G_n - G\, \phi_n)\, da \;, \tag{2.8}$$

where the notation $\nu \equiv \nu(\vec{x}) \equiv \vec{i} \cdot \vec{n}(\vec{x})$, $q \equiv q(\vec{x})$, and $p_x^F \equiv \partial_x p_F(x,y)$ was used.

Let ϕ in the integrand of the first integral on the left side of equation (2.8) be expressed as $\phi = \phi_0 + (\phi - \phi_0)$, where ϕ_0 is meant for $\phi(\vec{x}_0)$. Let also $\phi\, G_z + G\, \phi_{xx}$ in the fourth integral on the right side be expressed in the form $\phi\, G_z + G\, \phi_{xx} = \phi_0(G_z + G_{xx}) + (\phi - \phi_0)(G_z + G_{xx}) + (G\, \phi_x - \phi\, G_x)_x$. Equation (2.8) then becomes

$$\phi_0 \int_{d'} \nabla^2 G \, dv - \int_{d'} G \, q \, dv = \int_h \phi \, G_n \, da + \int_h G \, \nu \, da + \int_h G \, q \, da \; +$$

$$+ \; \phi_0 \int_{f'} (G_z + G_{xx}) \, dxdy + \oint_\gamma (G\phi_x - \phi G_x) \, dy + \oint_c (\phi G_x - G\phi_x) dy \; +$$

$$+ \int_{f'} G \, p_x^F \, dxdy + \int_{f'} G \, q \, dxdy + \int_\sigma (\phi G_n - G\phi_n) \, da - J \quad , \tag{2.9}$$

where (c) and (γ) are the intersection curves of (h) and (σ), respectively, with the plane $z = 0$, as it is shown in Figure 1, the Green relation

$$\int_{f'} (G\phi_x - \phi G_x)_x \, dxdy = \oint_\gamma (G\phi_x - \phi G_x) \, dy - \oint_c (G\phi_x - \phi G_x) \, dy$$

was used, and J is defined as

$$J = \int_{d'} (\phi - \phi_0)\nabla^2 G \, dv - \int_{f'} (\phi - \phi_0)(G_z + G_{xx}) \, dxdy \; .$$

We have $J \equiv 0$ by equations (2.5-2.6) and the fact that $\phi \to \phi_0$ as $\vec{x} \to \vec{x}_0$ [although ϕ could conceivably be discontinuous in (d'), e.g. across a vortex sheet, in a more advanced potential-flow model than the present one, in which no such discontinuities are allowed]. The surface integral over the surrounding surface (σ) and the line integral around the intersection curve (γ) of the surface (σ) with the plane $z = 0$ in equation (2.9) may be shown to vanish as the surrounding surface (σ) becomes ever larger. These two integrals may then be discarded, and we may in fact replace (f') by (f) and (d') by (d) in equation (2.9), which then becomes

$$C\phi_0 = \int_f G \, p_x^F \, dxdy + \int_h G \, \nu \, da + \int_h \phi \, G_n \, da + \oint_c (\phi G_x - G\phi_x) \, dy \; +$$

$$+ \int_h G \, q \, da + \int_f G \, q \, dxdy + \int_d G \, q \, dv \; , \tag{2.10}$$

where C is defined as

$$C = \int_d \nabla^2 G \, dv - \int_f (G_z + G_{xx}) \, dxdy \; . \tag{2.10a}$$

It may be seen from equations (2.5-2.6) that we have $C = 1$ for points $\vec{x}_0$ in the solution domain, excluding the fictitious ship hull surface (h) + (c), that is for points $\vec{x}_0$ in the domain [(d) + (f) - (h) - (c)], while we have $C = 0$ for points $\vec{x}_0$ "inside the ship", that is for points $\vec{x}_0$ in the domain [$(d_i) + (f_i)$ - (h) - (c)], where (d_i) is the domain and (f_i) the portion of the plane $z = 0$ which are inside the hull surface (h) + (c), as it is shown in Figure 1.

Equation (2.10) for $\vec{x}_0$ in [(d) + (f) - (h) - (c)] and with $C = 1$, or rather, in fact, the equation

$$\phi_0 = \int_h G\, \nu\, da + \int_h \phi\, G_n\, da + \oint_c (\phi G_x - G \phi_x)\, dy \tag{2.11a}$$

corresponding to the case of a displacement ship ($p_F = 0$) with the various correction terms q (which account for free-surface nonlinearities and effects of sinkage of trim, and may also represent "real-fluid effects", as it was explained in Part 1 of this study) being ignored, is well known, and is indeed often used for the purpose of expressing the disturbance velocity potential $\phi(\vec{x}_0)$ at points $\vec{x}_0$ of the flow domain — strictly outside the ship hull surface — in terms of the value of ϕ on the hull surface (h) + (c). Equation (2.10) with $C = 0$, that is for points $\vec{x}_0$ inside the ship (but not on the hull surface), or rather the equation

$$0 = \int_h G\, \nu\, da + \int_h \phi\, G_n\, da + \oint_c (\phi G_x - G \phi_x)\, dy \tag{2.11b}$$

corresponding to equation (2.11a), is also well known. It may be interesting to note that the method which is usually used for deriving equations (2.11a, b) is different from that used in the present study, although both methods certainly start from the same well-known Green identity (2.7). However, in the usual method of derivation of equation (2.11a) for $\vec{x}_0$ in (d), excluding (h) + (c), the Green identity (2.7) is applied to the domain [(d) - (ε)], where (ε) — by definition — represents the interior domain of a small sphere centered at $\vec{x}_0$; on the right side of equation (2.7), one then obtains an additional surface integral, over the boundary of (ε), whose limit value as the radius of the sphere vanishes is equal to $(-\phi_0)$. No such "limit process" was used (required) in the present study, where the functions G and ϕ are treated as generalized functions, as it is implied by equations (2.5) and (2.6).

It may be seen from equations (2.5-2.6) and equation (2.10a) that when $\vec{x}_0$ is right on the hull surface (h) + (c) we have $C = 1/2$, at least at points $\vec{x}_0$ where

(h) is smooth [more generally, the value of $4\pi C$, or $2\pi C$, at a point $\vec{x}_0$ of (h), or (c), is equal to the angle at which the domain (d), or (f), respectively, is viewed from the point $\vec{x}_0$]. Equation (2.10) for $\vec{x}_0$ on (h) + (c) and with C = 1/2, or rather the equation

$$\frac{1}{2}\,\phi_0 = \int_h G\,\nu\,da + \int_h \phi\,G_n\,da + \oint_c (\phi G_x - G\phi_x)\,dy \tag{2.11c}$$

corresponding to equations (2.11a,b), is also well known. Indeed, equation (2.11c) is the classical integral equation for the (linearized) Neumann-Kelvin problem, which is given, for instance, in Wehausen [6] p. 151 equation (3.55). As a matter of fact, equation (2.11c) for $\vec{x}_0$ on the hull surface, or analogous equations for similar potential-flow problems, corresponds to the equation one usually has in mind as an integral equation for potential-flow problems; indeed, Stoker [15] p. 195 refers to the (traditional) method by which equation (2.11c) for $\vec{x}_0$ on the body surface may be derived from the Green identity (2.7) as "the standard way of obtaining an integral equation for a harmonic function satisfying various boundary conditions". It will however be noted that it is only under the <u>assumption</u> of a <u>linearized</u> free-surface boundary condition, in which the nonlinear terms in equation (1.14a) are ignored, that equation (2.10) for $\vec{x}_0$ on the hull surface, and with C = 1/2, actually becomes an integral equation for the potential $\phi(\vec{x}_0)$ at the ship hull surface: clearly, knowledge of ϕ on (f) — where equation (2.10) with C = 1/2 however is not valid — is required in a nonlinear theory. For a <u>nonlinear</u> theory of steady motion of ships, <u>both</u> equations (2.10) with C = 1/2 (assuming a smooth hull surface) and C = 1 would therefore be required, and it would in fact be necessary to use each of these equations in turn for determining $\phi(\vec{x}_0)$ on (h) + (c) and in (d). This would lead to a fairly complicated numerical procedure in practice, which suggests that an alternative approach might be desirable; such an approach is proposed in the present study.

This new approach is based upon the obtention of a <u>single</u> equation equivalent to the system of two equations consisting of equations (2.10) with C = 1/2, valid for $\vec{x}_0$ on (h) + (c), and with C = 1, valid for $\vec{x}_0$ in (d) + (f) - (h) - (c). A straightforward derivation of this single equation equivalent to equations (2.10) with C = 1/2 and C = 1 is suggested by the fact that one must obviously "remedy" to the discontinuity of the value of the constant C at the ship hull surface [this discontinuity is evidently accompanied by a corresponding discontinuity of the values of the last two integrals on the right sides of equations (2.11), associated with the doublet distributions on the hull surface (h) and along the waterline (c)].

The discontinuity in the value of the constant C at (h) + (c) may readily be "remedied" by adding the expression $C_i\phi_0$, where C_i is defined as

$$C_i = \int_{d_i} \nabla^2 G \, dv - \int_{f_i} (G_z + G_{xx}) \, dxdy \ , \tag{2.12}$$

on both sides of equation (2.10), which then becomes

$$(C + C_i)\phi_0 = \int_f G \, p_x^F \, dxdy + \int_h G \, \nu \, da + \int_h \phi \, G_n \, da + \oint_c (\phi G_x - G\phi_x) \, dy + \int_h G \, q \, da +$$
$$+ \int_f G \, q \, dxdy + \int_d G \, q \, dv + \phi_0 \left[\int_{d_i} \nabla^2 G \, dv - \int_{f_i} G_z \, dxdy - \int_{f_i} G_{xx} \, dxdy \right] . \tag{2.13}$$

Indeed, we have

$$C + C_i = \int_{d + d_i} \nabla^2 G \, dv - \int_{f + f_i} (G_z + G_{xx}) \, dxdy$$

by equations (2.10a) and (2.12), and equations (2.5-2.6) show that $C + C_i = 1$ no matter whether the point $\vec{x}_0$ is in (d), on (f), on (h), on (c), or even in (d_i) or on (f_i). By using the "divergence theorem"

$$\int_{d_i} \nabla^2 G \, dv = - \int_h G_n \, da + \int_{f_i} G_z \, dxdy \ ,$$

and the Green relation

$$\int_{f_i} G_{xx} \, dxdy = \oint_c G_x \, dy$$

into equation (2.13), we finally obtain

$$\phi_0 = \int_f G \, p_x^F \, dxdy + \int_h G \, \nu \, da + \int_h (\phi - \phi_0) \, G_n \, da + \oint_c [(\phi - \phi_0)G_x - G\phi_x] \, dy +$$
$$+ \int_h G \, q \, da + \int_f G \, q \, dxdy + \int_d G \, q \, dv \ . \tag{2.14}$$

Equation (2.14) is an integral equation which may be used for determining the disturbance velocity potential $\phi(\vec{x}_0)$ in the solution domain (d), including its boundary (h) + (c) + (f), by means of a solution procedure based on iterations, as it will be explained in detail further on. Two essential features of the foregoing new integral equation are that it includes nonlinear free-surface effects, and that it is valid for points $\vec{x}_0$ on the hull surface (h) + (c) as well as "outside" (h), as it has already been noted. Other interesting features of the integral equation (2.14), which is the "key result" of the present study, will be pointed out further on.

Equation (2.14) is valid not only for $\vec{x}_0$ in (d) + (f) + (h) + (c), but also for $\vec{x}_0$ in $(d_i) + (f_i)$, that is for $\vec{x}_0$ "inside the ship". The case when $\vec{x}_0$ is "inside the ship" may be worth further consideration. As it was noted in connection with equation (2.10a) we have $C = 0$ in this case, which evidently implies that the expression on the right side of equation (2.10) is equal to zero, as it is indeed shown explicitly in equation (2.11b). This then shows that the integral equation (2.14) must actually take the form $0.\phi_0 = 0$ for $\vec{x}_0$ inside the ship. This integral equation therefore allows any arbitrary extension of the potential ϕ_0 outside its domain of definition (d). Conversely, no knowledge of ϕ_0 outside (d) and its boundary (h) + (f) is required in equation (2.14). These results are in agreement with what one would expect on the basis of plain common sense, for it would indeed be surprising, to say the least, if information given in the domain (d) and on its boundary — namely the Poisson equation (1.16) and the boundary conditions (1.14) and (1.15), together with the radiation condition — were sufficient for defining ϕ_0 (and hence a flow field associated with this potential) inside the ship, that is outside (d); conversely it would be equally surprising if information from outside (d) were required for defining the flow field in (d).

It will be noted that equation (2.10) with $C = 1$, that is for $\vec{x}_0$ in the solution domain (d) strictly "outside" the hull surface (h) + (c), may readily be obtained from the integral equation (2.14) by merely ignoring the term ϕ_0 in the expression $\phi - \phi_0$ in the doublet distributions, that is in the 3rd and 4th integrals, in equation (2.14). That the integral equation (2.14), unlike equation (2.10), is valid for $\vec{x}_0$ in (d) including (h) + (c), is evidently related to the fact that the values of the integrals involving the doublet distributions — of density $(\phi - \phi_0)$ — on (h) and (c) are <u>continuous</u> at (h) + (c), unlike the corresponding integrals — with doublet density ϕ — in equation (2.10), which are <u>discontinuous</u> at (h) or (c), as it is well known and was indeed already noted above.

The integral equation for surface-effect vessels

In this section we examine the integral equation (2.14) in the particular case of a surface-effect ship supported by an air cushion or a captive air bubble. In this case, the integral equation (2.14) takes the much-simplified form

$$\phi_0 = \iint_f G\, p_x^F \, dxdy + \iint_f G\, q \, dxdy \ , \tag{2.15}$$

where (f) represents the portion of the plane $z = 0$ where the free-surface pressure p_F is acting, and q is the correction associated with free-surface nonlinearities and wavebreaking defined by equation (1.14a) as

$$q = \left[\phi_z + \phi_{xx} + (|\nabla\phi|^2)_x + \tfrac{1}{2}\nabla\phi\cdot\nabla|\nabla\phi|^2 + \phi_x\, p_x^F + \phi_y\, p_y^F\right]_{z = -\phi_x - \frac{1}{2}|\nabla\phi|^2 - p_F} - \left[\phi_z + \phi_{xx}\right]_{z = 0} + q_{\text{wavebreaking}} \ . \tag{2.15a}$$

The integral equation (2.15), which obviously is a nonlinear integral equation in view of equation (2.15a), may be solved in a straightforward manner, at least in principle, by using a solution method based on iterations beginning with the "initial approximation" ϕ_{initial} obtained from equation (2.15) by ignoring the correction term q, and taking p_F as the given initial approximation $p_F^{(1)}$ to the pressure p_F acting on the free surface (the influence of the shape of the free surface upon p_F is neglected in the approximation $p_F^{(1)}$). We then have

$$\phi_{\text{initial}}(\vec{x}_0) = \iint_f G(\vec{x}_0,x,y,0)\ \partial_x p_F^{(1)}(x,y)\ dxdy \ . \tag{2.16}$$

It may be worthwhile to note in passing that an alternative expression for the "initial potential" ϕ_{initial} is

$$\phi_{\text{initial}}(\vec{x}_0) = \iint_f G_{x_0}(\vec{x}_0,x,y,0)\ p_F^{(1)}(x,y)\ dxdy \ , \tag{2.16a}$$

as it can be obtained from equation (2.16). It may be seen that the "initial approximation" ϕ_{initial} is actually identical with the "first approximation" $\phi^{(1)}$ defined in Part 1 as the solution of equations (1.17a), (1.17c), and the radiation condition.

It is also interesting to note that the approximation ϕ_{initial} corresponds to the first-order (linearized) approximation in a systematic perturbation scheme based on the assumption of a "small free-surface pressure distribution" p_F (see, e.g., Wehausen [6] p. 141).

The "first approximation" $\phi^{(1)} \equiv \phi_{\text{initial}}$ can then be used to evaluate the correction due to free-surface nonlinearities in equations (2.15a) and (2.15). This yields the "second approximation" $\phi^{(2)}$ defined as

$$\phi^{(2)}(\vec{x}_0) = \phi^{(1)}(\vec{x}_0) + \iint_f G(\vec{x}_0,x,y,0)q^{(1)}(x,y)\,dxdy \,, \tag{2.17}$$

with the first approximation $q^{(1)}$ to the nonlinear free-surface correction term q given by

$$q^{(1)}(x,y) = \left(p_F^{(1)}\right)_x + \Big[\phi_z^{(1)} + \phi_{xx}^{(1)} + \left(|\nabla\phi^{(1)}|^2\right)_x + \tfrac{1}{2}\nabla\phi^{(1)}\cdot\nabla|\nabla\phi^{(1)}|^2 + \phi_x^{(1)}\left(p_F^{(1)}\right)_x + \phi_y^{(1)}\left(p_F^{(1)}\right)_y\Big]_{z = -\phi_x^{(1)} - \frac{1}{2}|\nabla\phi^{(1)}|^2 - p_F^{(1)}} \tag{2.17a}$$

where equation (1.17c) was used, and the term $q_{\text{wavebreaking}}$ was ignored. This iterative procedure may be pursued, in principle, until the difference between successive approximations to ϕ is less than the desired accuracy; naturally, the influence of the shape of the free surface upon the free-surface pressure distribution p_F should be accounted for. The term $q_{\text{wavebreaking}}$ in equation (2.15a), which is suggested here as a (semi-empirical) ad hoc correction for approximately taking into account effects of wavebreaking (in the event wavebreaking does occur) in the framework of potential-flow theory, could, in principle, also be incorporated into the iterative scheme.

For most practical applications, the "second approximation" $\phi^{(2)}$ [or perhaps even the (linearized) "first approximation" $\phi^{(1)}$] may be sufficient. A notable simplification, which may be sufficient in practice, of the first approximation to the nonlinear free-surface correction term $q^{(1)}$ (and consequently to the "second approximation" $\phi^{(2)}$) may readily be obtained by expanding the expression on the right side of equation (2.17a) in a Taylor series about the plane $z = 0$, and retaining only "second-order terms", that is terms quadratic in ϕ , keeping in mind that we have $\phi^{(1)} = O(p_F^{(1)})$ in view of equation (2.16); we obtain

$$q^{(1)}(x,y) \simeq \left(|\nabla\phi^{(1)}|^2\right)_x - \left(\phi_x^{(1)} + p_F^{(1)}\right)\left(\phi_z^{(1)} + \phi_{xx}^{(1)}\right)_z + \phi_x^{(1)}\left(p_F^{(1)}\right)_x + \phi_y^{(1)}\left(p_F^{(1)}\right)_y \, , \quad (2.17b)$$

where the expression on the right side is to be evaluated on $z = 0$. The "second approximation" $\phi^{(2)}$ defined by equation (2.17), with the nonlinear free-surface correction term $q^{(1)}$ given by equation (2.17b), is identical to the second-order approximation in the systematic perturbation scheme for a "small free-surface pressure distribution" mentioned previously, as it may be verified.

This "small free-surface pressure distribution" perturbation scheme could naturally be pursued, in principle, beyond the second-order approximation defined by equations (2.17) and (2.17b), thereby providing an alternative method of solution of the problem of steady motion of a surface-effect ship. An appealing feature of this perturbation approach is that the successive approximations $q^{(k)}$ to the nonlinear free-surface correction term q , like the first approximation $q^{(1)}$ given in equation (2.17b), only require evaluation of $\phi(\vec{x}_0)$ on the plane $z_0 = 0$, so that the computational problem becomes two-, rather than three-, dimensional; this feature represents a notable advantage of the perturbation method in comparison with the iterative procedure, which requires, in effect, that $\phi(\vec{x}_0)$ be evaluated in a free-surface layer defined by $0 \geq z_0 \geq - \delta$, where δ must be so chosen as to permit the numerical extrapolation of ϕ and its derivatives above the plane $z_0 = 0$ required for evaluating the first (and subsequent) approximation to the free-surface nonlinear correction term $q^{(1)}$ given in equation (2.17a). A serious inconvenience of the perturbation approach, however, resides in that the complexity of the expressions for the higher-order approximations $q^{(k)}$ to the free-surface nonlinear correction term q increases very rapidly with k ; in this respect, the iterative procedure, in which the expression for the term q defined by equation (2.15a) remains the same at each step of the iterations is clearly at an advantage. In summary, it may be seen that while the iterative procedure and perturbation method are evidently very similar, they are not entirely identical; in particular, the two alternative methods of solution generate different sequences of successive approximations $\phi^{(k)}$, and there are also differences in the numerical implementation of these alternative methods.

The integral equation for displacement ships

We now consider the particular case of a displacement ship, for which the integral equation (2.14) takes the form

$$\phi_0 = \int_h G\,\nu\, da + \int_h (\phi - \phi_0) G_n\, da + \oint_c [G\phi_x - (\phi - \phi_0)G_x]\,\mu\, ds + \int_f G(q_{nl} + q_{wb})\, dxdy +$$
$$+ \int_h G(q_{hf} + q_{bl})\, da + \oint_c Gq_s\, ds + \int_d Gq_w\, dv \;, \qquad (2.18)$$

in which we used the relation $dy = -\mu\, ds$, where ds is the differential element of arc length along the "waterline" (c) and μ is defined as $\mu \equiv \vec{n}' \cdot \vec{i}$ with $\vec{n}'$ representing the unit inward normal to (c) in the plane z = 0; in the integral equation (2.18), q_{nl} represents the "NonLinear free-surface correction" term, which is given by equation (1.14a) as

$$q_{nl} = \left[\phi_z + \phi_{xx} + (|\nabla\phi|^2)_x + \tfrac{1}{2}\nabla\phi\cdot\nabla|\nabla\phi|^2\right]_{z = -\phi_x - \frac{1}{2}|\nabla\phi|^2} - \left[\phi_z + \phi_{xx}\right]_{z = 0} \;, \qquad (2.19)$$

q_{hf} is the "Hull-Form correction" term, which is associated with the fact that the "fictitious hull" (h) may differ from the actual ship hull (H), and is given by equation (1.15a) as

$$q_{hf} = (\vec{i} + \nabla\phi)_H \cdot \vec{N} - (\vec{i} + \nabla\phi)_h \cdot \vec{n} \;, \qquad (2.20)$$

q_{bl} is the correction associated with the viscous Boundary Layer around the ship hull, q_s is the correction associated with Spray formation, which was expressed as a line integral along the "waterline" (c), and q_{wb} and q_w are correction terms associated with WaveBreaking and viscosity effects in the Wake, respectively.

It will be noted that the term ϕ_x in the first line integral along (c) in equation (2.18) may be expressed in the form $\phi_x = \phi_s\, \vec{s} \cdot \vec{i} + \phi_t\, \vec{t} \cdot \vec{i} + \phi_n\, \vec{n} \cdot \vec{i}$, where $\vec{s}$ is the unit tangent vector to the "waterline" (c) oriented in the counterclockwise direction in the (x,y) plane, $\vec{n}$ is the unit normal vector to (h) pointing toward the interior of the ship (as it was defined previously), and $\vec{t}$ is the unit vector tangent to (h) mutually orthogonal to $\vec{s}$ and $\vec{n}$ and pointing downwards. From the hull condition (1.15) we have $\phi_n \equiv \nabla\phi \cdot \vec{n} \equiv -\vec{i} \cdot \vec{n} - q \equiv -\nu - q$, so that we have $\phi_x = \phi_s\sigma + \phi_t\tau - \nu^2 - q\nu$, where σ and τ are defined as $\sigma \equiv \vec{s} \cdot \vec{i}$ and $\tau \equiv \vec{t} \cdot \vec{i}$, and the integral equation (2.18) may then be expressed in the form

$$\phi_0 = \int_h G\,\nu\,da - \oint_c G\,\nu^2\mu\,ds + \int_h (\phi - \phi_0)G_n\,da + \oint_c [G(\sigma\phi_s + \tau\phi_t) - (\phi - \phi_0)G_x]\,\mu\,ds +$$

$$+ \int_f G(q_{nl} + q_{wb})dxdy + \int_h G(q_{hf} + q_{bl})da - \oint_c G[(q_{hf} + q_{bl})\nu\mu - q_s]ds + \int_d Gq_w dv \;. \quad (2.21)$$

It may be noted here that in the usual case when (h) intersects the plane $z = 0$ orthogonally we have $\tau \equiv \vec{t} \cdot \vec{i} = 0$ and $\nu \equiv \mu$ along the "waterline" (c), while in general we have $\nu = \mu\cos\gamma$ along (c) if γ is the angle between the normal $\vec{n}'$ to (c) and the normal $\vec{n}$ to (h) at (c). It may also be interesting to note that the factor $\sigma\mu$ multiplying the term ϕ_s in the second line integral in equation (2.21) is given by $\sigma\mu = -(\sin2\alpha)/2$, where α is the local angle between the x axis and the tangent vector $\vec{s}$ to the "waterline" (c), so that we have $0 \le |\sigma\mu| \le 1/2$, and in particular $\sigma\mu \to 0$ both as $\alpha \to 0$ and as $|\alpha| \to 90°$. On the other hand, the source density $\nu^2\mu$ in the first line integral is given by $\nu^2\mu = -\cos^2\gamma\sin^3\alpha$, so that we have $0 \le \nu^2|\mu| \le 1$, and in particular $\nu^2\mu \to 0$ as $\alpha \to 0$ and $\nu^2|\mu| \to 1$ as $|\alpha| \to 90°$ and $\gamma \to 0$. At the bow of a vertical-sided blunt ship $(\gamma = 0, \alpha = -90°)$ we thus have $\nu^2|\mu| >> |\sigma\mu|$.

The integral equation (2.21) may easily be seen to hold also in the cases of fully-submerged bodies, and multihull vessels (catamaran, trimaran, SWATH) with only few minor modifications: in the case of a fully-submerged body, (h) becomes a closed surface in the lower-half space $z < 0$, (f) becomes the whole plane $z = 0$, and the line integrals along the "waterline" (c) must be ignored, while in the case of a multihull vessel, a catamaran say, (h) and (c) become $(h_1) + (h_2)$ and $(c_1) + (c_2)$, respectively, and (d) and (f) become the portions of the lower-half space $z < 0$ and plane $z = 0$ outside the twin hulls (h_1) and (h_2) of the catamaran, and their intersections (c_1) and (c_2) with the plane $z = 0$, respectively. The integral equations (2.21) and (2.15) thus encompass most existing ships, although problems associated with lift and cavitation for hydrofoils, and planing effects and spray formation for fast boats, clearly were not considered in this study.

A straightforward (at least in principle) method of solving the integral equation (2.21) consists in using a solution procedure based on iterations, beginning with the <u>initial approximation</u>, ϕ_I say, obtained by ignoring the unknown terms in equation (2.21), in the manner discussed in the previous section in the case of the integral equation (2.15) for surface-effect vessels. We thus have

$$\boxed{\phi_I(\vec{x}_0) = \int_h G(\vec{x}_0,\vec{x})\,\nu(\vec{x})\,da(\vec{x}) - \oint_c G(\vec{x}_0,x,y,0)\,\nu^2(s)\,\mu(s)\,ds} \quad (2.22)$$

where $\nu \equiv \vec{n} \cdot \vec{i}$ and $\mu \equiv \vec{n}' \cdot \vec{i}$ as it was defined previously. In the case of a fine ship, for which α is small, we have $\nu^2|\mu| << |\nu|$, and the line integral in expression (2.22) may be neglected in comparison with the surface integral. It is interesting that the velocity potential, ϕ_H say, defined by this surface integral, that is

$$\phi_H(\vec{x}_0) = \int_h G(\vec{x}_0,\vec{x})\ \nu(\vec{x})\ da(\vec{x}) \tag{2.23}$$

actually corresponds to the wave resistance formula proposed by Hogner [1] in 1932 as an "interpolation" between Michell's "thin-ship approximation" and a "flat-ship approximation" — also proposed by Hogner in [1] — analogous to Michell's approximation. Indeed, these "thin- and flat-ship approximations", and also the "slender-ship approximation of Maruo [16], Tuck [17], and Vossers [18], can readily be derived from the Hogner wave resistance formula corresponding to expression (2.23) for the Hogner potential ϕ_H by means of appropriate assumptions, as it will be shown in detail in Part 3 of this study. The initial approximation ϕ_I therefore ought to provide a realistic initial approximation for solving the integral equation (2.21) iteratively if the ship hull form is sufficiently slender. At the bow of a vertical-sided blunt ship, we have $\nu^2\mu = \nu = 1$, so that we have no a priori reason to neglect the line integral in comparison with the surface integral (Hogner potential) in formula (2.22). As a matter of fact, it will be shown in Part 3 of this study that the line integral in equation (2.22) causes a drastic reduction in the value of the wave resistance at low Froude number.

The initial approximation ϕ_I (the notation ϕ^I will also be used wherever it is more convenient) can be used to define a second approximation ϕ_2 by evaluating the various terms which have previously been neglected in the integral equation (2.21); we thus obtain

$$\phi_2(\vec{x}_0) = \phi_I(\vec{x}_0) + \int_h (\phi^I - \phi_0^I)G_n\ da + \oint_c [G(\sigma\phi_s^I + \tau\phi_t^I) - (\phi^I - \phi_0^I)G_x]\ \mu\ ds +$$
$$+ \int_f Gq_{nl}^I\ dxdy + \int_h Gq_{hf}^I\ da - \oint_c Gq_{hf}^I\ \nu\mu\ ds \quad , \tag{2.24}$$

where the nonlinear free-surface correction term q_{nl}^I is given by

$$q_{nl}^I = \left[\phi_z^I + \phi_{xx}^I + (|\nabla\phi_I|^2)_x + \tfrac{1}{2}\nabla\phi_I\cdot\nabla|\nabla\phi_I|^2\right]_{z = -\phi_x^I - \frac{1}{2}|\nabla\phi_I|^2} \tag{2.24a}$$

since we have $\phi_z^I + \phi_{xx}^I = 0$ on $z = 0$, and the hull-form correction term q_{hf}^I is given by equation (2.20) with ϕ replaced by ϕ_I. A noteworthy simplification of expression (2.24a) for the nonlinear free-surface correction term q_{nl}^I is that obtained by using a Taylor series expansion about the plane $z = 0$ and retaining only the "second-order terms", that is the terms quadratic in ϕ or its derivatives; this yields

$$q_{nl}^I \simeq (|\nabla\phi_I|^2)_x - \phi_x^I\,(\phi_z^I + \phi_{xx}^I)_z\,, \tag{2.24b}$$

where the expression on the right side is to be evaluated on (f), that is on $z = 0$. It may be noted in passing that expression (2.24b) is identical to the expression obtained in the classical "second-order thin-ship theory", as it may be verified, for instance, from equation (3.27) in Noblesse and Dagan [4].

The second term on the right side of equation (2.24) represents the velocity potential of the flow, at point $\vec{x}_0$, due to a doublet distribution on the hull surface (h) of strength equal to the potential difference $\phi(\vec{x}) - \phi(\vec{x}_0)$, and thus corresponds to "interaction effects" of the hull upon itself; this doublet distribution might then conveniently be referred to as the "hull-interaction correction". The third term on the right side of equation (2.24) is a line integral along the "waterline" (c), so that this term might be referred to as the "waterline correction". An alternative expression for the "hull-interaction correction" and the "waterline correction" will be given below. The physical significance of the fourth term on the right side of equation (2.24) is clear: it accounts for both the nonlinear terms in the free-surface boundary condition and the difference between the actual free surface and the undisturbed free surface $z = 0$, where the free-surface condition is enforced for mathematical simplicity; this integral may thus be referred to as the "nonlinear free-surface correction". The last two terms on the right side of eq. (2.24) account for the possible difference between the actual hull surface (H) and the fictitious hull surface (h) where the hull boundary condition is enforced. In most practical applications, these terms would be associated with the variations in hull form due to the sinkage and trim experienced by the ship, so that evaluation of these "hull-form corrections" would evidently require preliminary determination of sinkage and trim. However, the "hull-form correction" q_{hf} could also be used in the "hull-design process" as a means for systematically investigating hull-form modifications about some "preliminary-design hull form" taken as the fictitious hull (h). Naturally, the "hull-form correction" vanishes if (h) coincides with (H), as it may be seen from equation (2.20). Although the terms q_{bl}, q_s, q_{wb},

and q_w in equation (2.21) were ignored in expression (2.24) for the second approximation ϕ_2, these ad hoc corrections for "real-fluid effects", which might be referred to as the "boundary layer, spray, wavebreaking, and wake corrections", respectively, could in principle be included.

The "hull-interaction correction" and the "waterline correction" in expression (2.24) for the second approximation ϕ_2 involve distributions of both sources and doublets. An alternative expression for these correction terms — involving distributions of sources alone — can be obtained, as it will now be shown. We begin by considering a function ψ verifying the Laplace equation $\nabla^2\psi = 0$ in (d), the linearized "Neumann-Kelvin condition" $\psi_z + \psi_{xx} = 0$ on (f), the "Neumann condition" $\psi_n = \psi_n$ on the hull surface (h), and the "radiation condition" of "no waves upstream". By comparing the above equations with equations (1.16), (1.14), and (1.15), respectively, it may be shown that the integral equation (2.21) takes the form

$$\psi_0 = -\int_h G\,\psi_n\,da + \oint_c G\,\psi_n\,\nu\mu\,ds + \int_h (\psi - \psi_0)G_n\,da +$$
$$+ \oint_c [G(\sigma\psi_s + \tau\psi_t) - (\psi - \psi_0)G_x]\,\mu\,ds \quad . \tag{2.25}$$

By using equation (2.25), with ψ taken as the initial potential ϕ_I, we may obtain the following expression for the "hull-interaction correction" and the "waterline correction" in expression (2.24)

$$\int_h (\phi^I - \phi_0^I)G_n\,da + \oint_c [G(\sigma\phi_s^I + \tau\phi_t^I) - (\phi^I - \phi_0^I)G_x]\,\mu\,ds =$$
$$\int_h G(\nu + {}^e\phi_n^I)\,da - \oint_c G(\nu + {}^e\phi_n^I)\,\nu\mu\,ds \,, \tag{2.26}$$

where formula (2.22) for ϕ_0^I was used, and the superscript "e" in the term ${}^e\phi_n^I$ is meant to clearly indicate that the term $\phi_n^I \equiv \nabla\phi_I \cdot \vec{n}$, which is discontinuous across (h) since the potential ϕ_I is associated with a distribution of sources on (h), must be evaluated on the "exterior" side of the surface (h). By substituting equation (2.26) into equation (2.24), we obtain the following alternative expression for the second approximation ϕ_2

$$\boxed{\phi_2(\vec{x}_0) = \phi_I(\vec{x}_0) + \int_h G(\nu + \phi_n^I)_H\,da - \oint_c G(\nu + \phi_n^I)_H\,\nu\mu\,ds + \int_f G q_{n1}^I\,dxdy} \tag{2.27}$$

where the surface and line integrals over (h) and along (c), respectively, have been grouped together, and expression (2.20) for the "hull-form correction term" q^I_{hf} was used[+].

Numerical evaluation of the second approximation ϕ_2 defined by equation (2.27) may be divided into four basic steps, as follows: (i) evaluate the initial approximation ϕ_I given by formula (2.22) for some fictitious hull surface (h), which may -but need not- be taken as the wetted hull of the ship in position of rest, (ii) determine the sinkage and trim experienced by the ship and the position of the hull (H) corresponding to the approximation ϕ_I , (iii) evaluate the fluid flux $(\nu + \phi^I_n)_H \equiv [\vec{i} + (\nabla\phi_I)_H] \cdot \vec{N}$ across the ship hull (H) and the nonlinear free-surface correction flux q^I_{nl} , and finally (iv) evaluate the last three terms on the right side of expression (2.27). The computational task involved in the practical implementation of the above four basic steps admittedly is quite considerable, but it ought to be within present-day calculation capabilities. Although the iterative procedure described in the foregoing could in principle be pursued further, the enormous computational task involved in the continuation of this iterative scheme drastically limits the feasibility of this approach in practice. It may seem reasonable to hope, however, that the second approximation ϕ_2 , or perhaps even the initial approximation ϕ_I , may be sufficiently accurate for most practical applications.

Variations about the basic iterative scheme described above may naturally be considered. In particular, it may be interesting to begin, in a first stage, by seeking the "linearized Neumann-Kelvin potential", which was denoted by $\phi^{(1)}$ in equations (1.17) but will simply be denoted by φ here, given by the solution of the linearized integral equation

$$\varphi_0 = \phi^I_0 + \int_h (\varphi - \varphi_0)G_n \, da + \oint_c [G(\sigma\varphi_s + \tau\varphi_t) - (\varphi - \varphi_0)G_x] \, \mu \, ds \, , \tag{2.28}$$

in which the "nonlinear free-surface correction", the "hull-form correction", and the various "real-fluid effects" corrections, in the integral equation (2.21) have been neglected, and equation (2.22) was used. The main advantage of the linearized integral equation (2.28) in comparison with equation (2.21) is that it involves the potential φ on the fictitious hull surface (h) + (c) alone so that the solution φ of the linearized Neumann-Kelvin integral equation (2.28) may be sought on (h) + (c) alone [whereas equation (2.21) requires that ϕ be determined in some subdomain of (d) "outside" (h)]. The iterative scheme discussed previously may be used for solving

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[+]Equation (2.20), with ϕ replaced by ϕ_I, yields

$$q^I_{hf} = (\vec{i} + \nabla\phi_I)_H \cdot \vec{N} - (\vec{i} + \nabla\phi_I)_h \cdot \vec{n} = (\nu + \phi^I_n)_H - (\nu + \phi^I_n)_h$$

the integral equation (2.28); specifically, the sequence of approximations $\psi^{(k)}$, $k \geq 0$ defined by $\psi^{(0)} = 0$ and the recurrence relation

$$\psi_0^{(k+1)} = \phi_0^I + \int_h (\psi^{(k)} - \psi_0^{(k)})G_n \, da + \oint_c [G(\sigma\psi_s^{(k)} + \tau\psi_t^{(k)}) - (\psi^{(k)} - \psi_0^{(k)})G_x]\mu \, ds \quad (2.28a)$$

may be associated with the integral equation (2.28). By using equation (2.25), with ψ replaced by $\psi^{(k)}$, into equation (2.28a) we may obtain the following alternative recurrence relation

$$\psi_0^{(k+1)} = \psi_0^{(k)} + \int_h G(\nu + {}^e\psi_n^{(k)}) \, da - \oint_c G(\nu + {}^e\psi_n^{(k)}) \, \nu\mu \, ds \ , \ k \geq 0 \ , \quad (2.28b)$$

where equation (2.22) was used, and the notation ${}^e\psi_n^{(k)}$ means that the normal derivative $\psi_n^{(k)} \equiv \nabla\psi^{(k)} \cdot \vec{n}$ of the potential $\psi^{(k)}$ must be evaluated on the "exterior side" of the hull surface (h). Once equation (2.28) has been solved (within some prescribed accuracy) for ψ on (h) + (c), one may turn to the nonlinear integral equation (2.21) and seek to correct the "linearized Neumann-Kelvin potential" ψ by evaluating the "nonlinear free-surface correction" and the "hull-form (sinkage and trim) correction". Evaluation of these corrections for free-surface nonlinearities and effects of sinkage and trim evidently requires that ψ be determined "outside" the hull surface (h) + (c); this can be done by using the equation

$$\psi_0 = \phi_0^I + \int_h \psi \, G_n \, da + \oint_c [G(\sigma\psi_s + \tau\psi_t) - \psi G_x] \, \mu \, ds \ , \quad (2.29)$$

which may be obtained from equation (2.11a), and is valid for $\vec{x}_0$ strictly "outside" (h) + (c). Further improvement of this Neumann-Kelvin approximation corrected (approximately) for effects of free-surface nonlinearities and of sinkage and trim could, in principle, then be sought by pursuing the iterative method of solution of the nonlinear integral equation (2.21) described previously.

Another noteworthy variation about the first-discussed iterative scheme — which consists in using the potential ϕ_I given by equation (2.22) as initial approximation for solving the integral equation (2.21) — is the iterative scheme associated with the use of the Hogner potential ϕ_H given by equation (2.23) as initial approximation for solving the integral equation (2.18). The second approximation, ϕ_2' say, in this iterative scheme is given by

$$\phi_2'(\vec{x}_0) = \phi_H(\vec{x}_0) + \int_h (\phi^H - \phi_0^H) G_n \, da + \oint_c [G\phi_x^H - (\phi^H - \phi_0^H) G_x] \, \mu \, ds +$$
$$+ \int_f G q_{nl}^H \, dxdy + \int_h G q_{hf}^H \, da \, , \tag{2.30}$$

as it can readily be obtained from equation (2.18). It can be shown — in the manner used previously for deriving equation (2.27) — that expression (2.30) for ϕ_2' may also be written in the alternative form

$$\phi_2'(\vec{x}_0) = \phi_H(\vec{x}_0) + \int_h G(\nu + \phi_n^H)_H \, da + \int_f G q_{nl}^H \, dxdy \tag{2.30a}$$

which corresponds to expression (2.27) for the second approximation ϕ_2 , while expression (2.30) for ϕ_2' corresponds to expression (2.24) for ϕ_2 . It can also be shown that the solution, i.e φ , of the "linearized Neumann-Kelvin problem" can be defined as the limit of the sequence of approximations $\check{\varphi}^{(k)}$, $k \geq 0$, defined by $\check{\varphi}^{(0)} = 0$ and the recurrence relation

$$\check{\varphi}_0^{(k+1)} = \check{\varphi}_0^{(k)} + \int_h G(\nu + {}^e\check{\varphi}_n^{(k)}) \, da \, , \quad k \geq 0 \, , \tag{2.31}$$

which corresponds to the recurrence relation (2.28b) associated with the sequence of approximations $\varphi^{(k)}$. Comparison between expressions (2.22) and (2.23) for the initial approximations ϕ_I and ϕ_H , expressions (2.27) and (2.30a) for the second approximations ϕ_2 and ϕ_2' , and the recurrence relations (2.28b) and (2.31) for the sequences of approximations $\varphi^{(k)}$ and $\check{\varphi}^{(k)}$ show that the line integrals which appear in the iterative approximations associated with the use of ϕ_I as initial approximation do not occur in the iterative approximations associated with the use of the Hogner potential ϕ_H as initial approximation, so that the latter iterative approximations may perhaps be a little simpler to evaluate numerically than the former. The fact that there is no line integral in the iterative scheme corresponding to the recurrence relation (2.31) and approximations (2.23) and (2.30a) is in agreement with the result that no line integral occurs in the classical "thin-ship perturbation approximations", which can be expressed — at any order of approximation — in terms of surface distributions of sources on the ship centerplane and on the undisturbed free surface, as it is shown in Noblesse and Dagan [4]. This result thus is generalized in the iterative approximations (2.23) and (2.30a), which express the disturbance velocity potential ϕ in terms of

surface distributions of sources on some fictitious hull surface (h) and on the undisturbed free surface [higher iterative approximations ϕ_k' , $k \geq 3$, are also expressed in terms of surface distributions of sources on (h) and (f), as it can easily be shown].

As a matter of fact, the classical "thin-ship perturbation approximations", which may be derived from the usual differential formulation of the problem by performing a systematic perturbation analysis (in terms of the ship beam/length ratio as "small perturbation parameter"), may also be obtained from the new integral formulation of the problem given in this study, namely the integral equation (2.18), by using a "thin-ship iterative solution procedure", which essentially corresponds to the "thin-ship limit" of the iterative scheme associated with the use of the Hogner potential ϕ_H as initial approximation, as it will now be shown. Effects of sinkage and trim will be ignored, and we assume that the equation of the wetted hull of the ship in position of rest — which is taken as the fictitious hull surface (h) — may be written in the form $y = \pm\, b(x,z)$. The surface integral over (h) in expression (2.23) for the Hogner potential ϕ_H can then be transformed into a double integral over the projection (h_y) of (h) onto the ship centerplane $y = 0$; we thus obtain

$$\phi_H(\vec{x}_0) = 2\iint_{h_y} G[\vec{x}_0,x,y=b(x,z),z]\; b_x(x,z)\; dxdz \;.$$

In the thin-ship limit, that is if $b(x,z) << 1$, the Hogner potential ϕ_H becomes the classical Michell potential, ϕ_M say, given by

$$\phi_M(\vec{x}_0) = 2\iint_{h_y} G(\vec{x}_0,x,y=0,z)\; b_x(x,z)dxdz \quad .$$

By replacing ϕ_H by ϕ_M in equation (2.30a), we obtain the second approximation, ϕ_2^* say, given by

$$\phi_2^*(\vec{x}_0) = \phi_M(\vec{x}_0) + 2\iint_{h_y} G(\vec{x}_0,x,y=b,z)\; \left[b_x - \phi_y^M + \phi_x^M b_x + \phi_z^M b_z\right]_{y\,=\,b} dxdz \; + $$
$$+ \iint_f G(\vec{x}_0,x,y,z=0)\; \left[(|\nabla\phi_M|^2)_x - \phi_x^M\,(\phi_z^M + \phi_{xx}^M)_z\right]_{z\,=\,0} dxdy \quad ,$$

where the surface integral over (h) was transformed into a double integral over (h_y), expression (2.24b) — with ϕ_I replaced by ϕ_M — for the nonlinear free-surface flux q_{nl}^M was used, and the nonlinear free-surface correction integral was explicitly written as a double integral for consistency. By using a Taylor series expansion about the ship centerplane $y = 0$, we may obtain

$$\left[b_x - \phi_y^M + \phi_x^M b_x + \phi_z^M b_z\right]_{y=b} = \left[(\phi_x^M b)_x + (\phi_z^M b)_z\right]_{y=0} + O(b^2)$$

since we have $\nabla^2 \phi_M = 0$ and $b_x = (\phi_y^M)_{y=+0}$. In the thin-ship limit, the second approximation $\overset{*}{\phi}_2$ defined above becomes the "second iterative thin-ship approximation", $\tilde{\phi}_2$ say, given by

$$\tilde{\phi}_2(\vec{x}_0) = \phi_M(\vec{x}_0) + 2\iint_{h_y} G(\vec{x}_0,x,y=0,z)\left[(\phi_x^M b)_x + (\phi_z^M b)_z\right]_{y=0} dxdz +$$

$$+ \iint_{z=0} G(\vec{x}_0,x,y,z=0)\left[(|\nabla\phi_M|^2)_x - \phi_x^M(\phi_z^M + \phi_{xx}^M)_z\right]_{z=0} dxdy ,$$

which is in agreement with the expression for the classical "second-order thin-ship perturbation approximation", as it may be verified by comparison with equations (3.16), (3.28), (3.27), and (4.5) in Noblesse and Dagan [4]. The iterative scheme associated with the use of the Hogner potential ϕ_H as initial approximation may thus be regarded as a "generalized thin-ship iterative scheme". It is however perhaps more appropriate to refer to this iterative scheme as the "fine-ship iterative scheme", which is in agreement with the fact that the line integral in expression (2.22) for the initial approximation ϕ_I may be neglected in comparison with the Hogner potential ϕ_H if the angle α between the "waterline" (c) and the x axis is sufficiently small, that is for a "fine ship"; the Hogner potential ϕ_H and the potential ϕ_2' given by equations (2.23) and (2.30a), respectively, may thus be regarded as the "first and second fine-ship (iterative) approximations", respectively.

It may finally be worthwhile to emphasize that the fictitious hull surface (h) may be chosen at will — at least to a certain extent — in the present theory, and we may naturally seek to take advantage of this freedom. In particular, expressions (2.27) and (2.30a) for the second approximations ϕ_2 and ϕ_2' suggest that an "equivalent Hogner hull" (h) which by definition would be such that the fluid flux $(\nu + \phi_n^H)_H \equiv [\vec{i} + (\nabla\phi_H)_H]\cdot\vec{N}$ across the actual ship hull surface (H) is zero (or sufficiently small) might be determined, say by means of some iterative procedure in which the shape of the fictitious hull (h) is varied systematically in some appropriate manner; thus, in this approach, iterations would be performed with regard to the shape of the fictitious hull surface (h), rather than with regard to the velocity potential ϕ as in the iterative schemes discussed previously. The idea of purposely choosing the fictitious hull surface (h) so that it differs from the actual ship hull surface (H) may also be useful outside this "concept of the equivalent Hogner hull surface" in that choosing (h) different from (H) would circumvent the numerical difficulties associated with the singular behavior of the Green function $G(\vec{x}_0,\vec{x})$ when the points $\vec{x}$ and $\vec{x}_0$ coincide.

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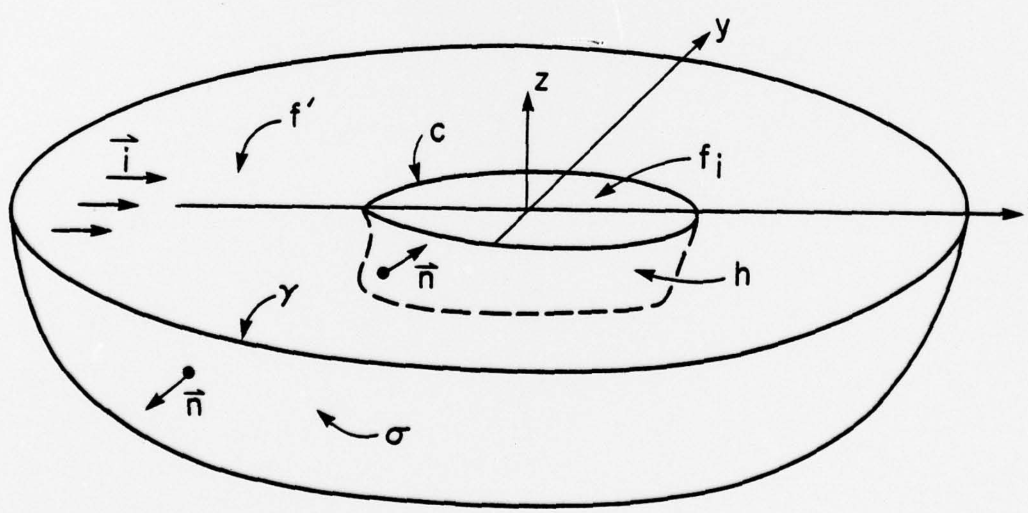

Figure 1: DEFINITION SKETCH